## Scheme / Structure of M.Sc Mathematics CBCS course for Department of Mathematics, K.U.K

<u>Course Name</u>: Master of Science in Mathematics

CBCS course for Department of Mathematics, KUK

**Course Duration**: Four Semesters

<u>Course Code</u> : MSM

To be effective: With effect from Session 2016-17 for 1<sup>st</sup> and 2<sup>nd</sup> Semesters and from

2017-18 for 3<sup>rd</sup> and 4<sup>th</sup> Semesters in the Department of Mathematics,

K.U. Kurukshetra

## General Rules :

1. There will be five theory papers and one practical paper in each semester. In addition, there will be two seminars, one each in  $1^{st}$  semester and  $4^{th}$  semester and two open elective papers, one each in  $2^{nd}$  and  $3^{rd}$  semesters.

- 2. A student of M.Sc Mathematics CBCS course shall have to opt one Open Elective Paper in second semester and one Open Elective Paper in third semester out of the list of Open Elective Papers offered at the level of Faculty of Sciences except those which are offered as Open Elective Papers by the Department of Mathematics.
- 3. Each theory paper (Core and Elective) will be of 100 marks, 70 marks for External Examination and 30 marks for Internal Assessment.
- 4. Each practical paper and open elective paper will be of 50 marks, 35 marks for External Examination and 15 marks for Internal Assessment.
- 5. Each seminar will be of 50 marks. The evaluation of seminars, which will be presented by every student in that semester, will be done by a Departmental Committee which will be constituted by the Staff Council of the Department and marks shall be awarded by the committee out of 50 marks. There shall be no external examination of the seminar.
- 6. Each theory paper will consist of two sections.
- 7. Paper setter will be requested to set eight questions in all, i.e., four questions from each section.
- 8. The examinee will be required to attempt five questions in all by selecting at least two questions from each section. All questions will be of equal marks.
- 9. Duration of examination of each theory paper will be of three hours and duration of examination of each practical paper will be of four hours.
- 10. The minimum pass percentage required to pass each paper will be as under:
  - i. 40% in each theory (C/E/OE) External Examination
  - ii. 40% in each Practical External Examination
  - iii. 40% in each Seminar
  - iv. 40% in aggregate of External Examination and Internal Assessment Test of each theory (C/E/OE) and Practical Paper.

- 11. The following criteria shall be adopted for the award of Internal Assessment of Minor tests for each paper:
  - i. Theory papers: -

a. Total Internal Assessment
b. Two class tests (each of 1 hour duration)
c. Attendance
d. 30 marks
22.5 marks
7.5 marks

ii. Practical papers: -

a. Total Internal Assessment
b. Seminar / Viva voce for each practical paper
c. Attendance
d. 15 marks
marks
5 marks

iii. Open elective theory papers: -

a. Total Internal Assessment
b. One class test (of 1 hour duration)
c. Attendance
d. 15 marks
i. 10 marks
j. 5 marks

iv. Criterion for the award of marks for attendance will be as follows:

Marks for attendance (Out of 7.5 marks) will be given as under:

a) 91 % onwards : 7.5 Marks b) 81 % to 90 % : 6 Marks c) 75 % to 80 % : 4.5 Marks d) 70 % to 74 % : 3 Marks\* e) 65 % to 69 % : 1.5 Marks\*

Marks for attendance (Out of 5 marks) will be given as under:

a) 91 % onwards : 5 Marks
b) 81 % to 90 % : 4 Marks
c) 75 % to 80 % : 3 Marks
d) 70 % to 74 % : 2 Marks\*
e) 65 % to 69 % : 1 Mark\*

- v. For practical papers, it will be optional for the department concerned to conduct either a viva-voce or a sessional test for each paper. The test of one hour duration for each paper will be conducted by the Department concerned at its own level.
- vi. Internal assessment test/viva-voce is compulsory. In case the student(s) remain absent from appearing in the test(s)/viva-voce, the Chairperson of the Department concerned will decide the case at his/her own level.
- vii. The marks of assessment/grades of Minor Test will be displayed on the notice board of the Department by the Chairperson of the Department before forwarding it to the Examination Branch.
- viii. The Chairperson of the Department shall forward the final awards/grade of Minor Tests of the students in hard and soft copy to the Examinations Branch invariably within 20 days after the completion of the relevant Semester examination for declaration of the result and preparation of Transcript/DMC. The Minor Test Grade of a candidate who fails in any semester examination shall be carried forward to the next examination.
- ix. The Chairperson of the Department shall preserve the record on the basis of which the Minor Test Grade has been prepared, for inspection, if needed by the University upto six months from the date of declaration of the concerned semester result.
- 12. One credit is equivalent to 25 marks.

<sup>\*</sup> For students engaged in co-curricular activities of the Department only / authenticated medical grounds duly approved by the concerned Chairperson.

13. Teaching hours : One credit is equivalent to one hour of teaching (lecture/

tutorial/seminar) per week per semester for each theory paper, One credit is equivalent to two hours of practical work per week

per semester.

Theory papers (Core or Elective): Four hours for lectures per week per paper

Practical paper : Four hours per week per paper for a group of

fifteen students.

Seminar : Two hours per week for a group of fifteen students

Theory Papers (Open Elective) : Two hours per week per paper

## 14. Abbreviations used:

C : Core Paper E : Elective Paper

OE : Open Elective Paper

L : Lecture
T : Tutorial
P : Practical
S : Seminar

# Scheme / Structure of M.Sc. Mathematics CBCS Four Semester Course for Department of Mathematics, KUK (w.e.f. Session 2016-17)

# Semester - I

Paper Code	C/E/OE	Name of Paper	Contact hours			Credits
			L	P	S	
<b>Core Papers</b>						
MSM 101	C	Abstract Algebra	4	0	0	4
MSM 102	С	Complex Analysis	4	0	0	4
MSM 103	С	Ordinary Differential Equations	4	0	0	4
MSM 104	C	Real Analysis	4	0	0	4
MSM 105	C	Topology	4	0	0	4
MSM 106	С	Practical-I	0	4	0	2
MSM 107	С	Seminar-I	0	0	2	2
		Total	20	4	2	24

**Semester - II** 

Paper Code	C/E/OE	Name of Paper	Contact hours			Credits
			L	P	S	
Core Papers						
MSM 201	C	Advanced Abstract Algebra	4	0	0	4
MSM 202	С	Computer Programming	4	0	0	4
MSM 203	C	Measure and Integration	4	0	0	4
MSM 204	С	Mechanics of Solids	4	0	0	4
MSM 205	С	System of Differential Equations	4	0	0	4
MSM 206	С	Practical-II	0	4	0	2
Open Elective Pa	apers					
	OE	One open elective paper is to be opted out of the list of optional papers offered at the Faculty of Science level in even semester	2	0	0	2
		Total	22	4	0	24
OE 207		Applied Algebra and Analysis	2	0	0	2
		(This open elective paper will be offered to the students of Faculty of Sciences except students of Department of Mathematics)				

Semester - III

Paper Code	C/E/OE	Name of Paper	<b>Contact Hours</b>		rs	Credits
			L	P	S	
<b>Core Papers</b>					I	1
MSM 301	С	Functional Analysis	4	0	0	4
MSM 302	С	Fluid Mechanics	4	0	0	4
MSM 303	С	Practical-III	0	4	0	2
<b>Elective Papers</b>		Any three of the following Elective l	Papers		I	1
MSM 304	Е	Advanced Topology	4	0	0	4
MSM 305	Е	Algebraic Coding Theory	4	0	0	4
MSM 306	Е	Commutative Algebra	4	0	0	4
MSM 307	Е	Differential Geometry	4	0	0	4
MSM 308	Е	Elasticity	4	0	0	4
MSM 309	Е	Financial Mathematics	4	0	0	4
MSM 310	Е	Fuzzy Sets and Applications	4	0	0	4
MSM 311	Е	Integral Equations	4	0	0	4
MSM 312	Е	Mathematical Modeling	4	0	0	4
MSM 313	Е	Mathematical Statistics	4	0	0	4
MSM 314*	Е	Methods of Applied Mathematics	4	0	0	4
MSM 315	Е	Number Theory	4	0	0	4
Open Elective Pa	aper					
	OE	One open elective paper is to be opted out of the list of optional papers offered at the Faculty of Science Level in odd Semester	2	0	0	2
		Total	22	4	0	24
OE 307		Applied Numerical Methods (This open elective paper will be offered to the students of Faculty of Sciences except students of Department of Mathematics)	2	0	0	2

<sup>\*</sup> Syllabus will be prepared later on.

# Semester - IV

Paper Code	C/E/OE	Name of Paper	<b>Contact Hours</b>		rs	Credits
			L	P	S	
Core Papers						
MSM 401	С	Mechanics and Calculus of Variations	4	0	0	4
MSM 402	С	Partial Differential Equations	4	0	0	4
MSM 403	С	Practical – IV	0	4	0	2
MSM 404	С	Seminar-II	0	0	2	2
<b>Elective Papers</b>		Any three of the following Elective	Papers	•	•	
MSM 405	Е	Advanced Complex Analysis	4	0	0	4
MSM 406	Е	Advanced Discrete Mathematics	4	0	0	4
MSM 407	Е	Advanced Functional Analysis	4	0	0	4
MSM 408	Е	Algebraic Number Theory	4	0	0	4
MSM 409	Е	Analytic Number Theory	4	0	0	4
MSM 410	Е	Bio-Mathematics	4	0	0	4
MSM 411	Е	Boundary Value Problems	4	0	0	4
MSM 412	Е	Fluid Dynamics	4	0	0	4
MSM 413	Е	General Measure and Integration Theory	4	0	0	4
MSM 414	Е	Linear Programming	4	0	0	4
MSM 415	Е	Mathematical Aspects of Seismology	4	0	0	4
MSM 416	Е	Non-Commutative Rings	4	0	0	4
MSM 417	Е	Wavelet Analysis	4	0	0	4
		Total	20	4	2	24

# LEARNING OBJECTIVES AND OUTCOMES OF DIFFERENT COURSES <u>SEMESTER - I</u>

#### MSM 101: ABSTRACT ALGEBRA

The concept of a group is surely one of the central ideas of mathematics. The main aim of this course is to introduce Sylow theory and some of its applications to groups of smaller orders. An attempt has been made in this course to strike a balance between the different branches of group theory, abelian groups, nilpotent groups, finite groups, infinite groups and to stress the utility of the subject. A study of Modules, submodules, quotient modules, finitely generated modules etc. is also promised in this course. Hilbert basis theorem and Wedderburn -Artin theorem are the heighlights of this course.

#### MSM-102: COMPLEX ANALYSIS

One objective of this course is to develop the parts of the theory that are prominent in applications of the complex numbers. Other objective is to furnish an introduction to applications of residues and conformal mapping. With regard to residues, special emphasis is given to their use in evaluating real improper integrals, finding inverse Laplace transforms, and locating zeros of functions. Conformal mapping find its use in solving boundary value problems that arise in studies of heat conduction, fluid flow and elastodynamics.

# MSM 103: ORDINARY DIFFERENTIAL EQUATIONS

This course has been framed to learn the theory of ordinary differential equations. Existence and uniqueness theory of solution of an ordinary differential equation and of an initial value problem is to be learnt during the course. Theory of homogeneous and non-homogeneous linear differential equations of higher order, Adjoint equations and Wronskian theory are also learnt during the course. Students will also learn second order ordinary differential equations and Sturm theory, Oscillation theory, boundary value problems and Greens functions in the context of such differential equations. On completion of the course, a student will be able to understand the theory of ordinary differential equations of 2nd and higher order and to know the techniques of solving them.

## MSM 104: REAL ANALYSIS

This course has been developed to introduce some fundamental topics of mathematical analysis which are directly relevant in some other papers of M.Sc. Mathematics course. In this course the students will be taught Riemann Stieltjes integral, uniform convergence of sequences and series of functions, and functions of several variables.

## **MSM 105: TOPOLOGY**

This course is a systematic exposition of the part of general topology which has proven useful in several branches of mathematics. Starting from the statements of Axiom of choice, Zorn's lemma, Well ordering theorem and Continuum hypothesis, we move on to the introduction of topological spaces and their properties. Some of the main topics taught in this course include Product and Quotient spaces, Embedding and Metrization, Compactness, Continuity and Filters.

## **MSM 106: PRACTICAL-I**

This course is in continuation to the paper on numerical methods that students study during their graduate course. ANSI-C programming makes a part of that paper but students restrict themselves only to the theoretical knowledge. Hence, the objective of this course is to acquaint the students with the practical use of ANSI-C, for solving some problems of social and mathematical kind. Also some problem solving techniques based on papers MSM 101 to MSM 105 will be taught.

## MSM 107: SEMINAR-I

In this course a student will learn to select the topic amongst syllabi of other courses prescribed in this semester. A student will learn to collect, review and to understand the literature and to present the contents of the topic so chosen. After the completion of this course the student will get an exposure towards self study and enhancement of presentation skills.

## SEMESTER - II

#### MSM 201: ADVANCED ABSTRACT ALGEBRA

As suggested by the name of the course itself, some of the advanced topics of abstract algebra will be taught to the students in this course including field extensions, finite fields, normal extensions, finite normal extensions as splitting fields. A study of Galois extensions, Galois groups of polynomials, Galois radical extensions shall also be made. Similar linear transformations, Nilpotent transformations and related topics are also included in the course.

## MSM 202: COMPUTER PROGRAMMING

This course is designed to train the students for complete knowledge of computer programming in FORTRTAN-90, along with the additional features of FORTRAN-95. The course enables the students to write any source program to compute the numerical solutions of the mathematics problems, which arise in the research studies with applications to engineering, physical, biological or social sciences.

## **MSM 203: MEASURE AND INTEGRATION**

One of the basic concepts of analysis is that of integration. The classical theory of integration has certain drawbacks: even relatively simple functions are not integrable in the Riemann sense. These deficiencies have been removed in the theory of Lebesgue measure and integration. This course aims at providing an introduction to the theory of Lebesgue measure and integration. The students will be taught Measurable Sets, Measurable functions, Lebesgue Integral, Differentiation and Integration and The Lebesgue L<sup>p</sup> Spaces.

#### MSM 204: MECHANICS OF SOLIDS

In this course, basic theory of mechanics of solids is introduced. First, the laws of transformations and tensors will be introduced. Mathematical theory of deformations, analysis of strain and analysis of stress in elastic solids will be learnt next. A student will also learn basic equations of elasticity and variational methods. In this course, the students will be exposed to the mathematical theory of elasticity and other techniques which find applications in areas of civil and mechanical engineering and Earth and material sciences. After completion of this course, a student will get enough exposure to Applied Mathematics and this course will form a sound basis for doing research in the number of areas involving solid mechanics.

## MSM 205: SYSTEM OF DIFFERENTIAL EQUATIONS

This course has been designed to understand system of differential equations including linear and non-linear systems. Initially, linear differential systems (homogeneous and non-homogeneous) and the existence and uniqueness theory for such systems are to be learnt and thereafter the theory is extended to system of n differential equations including non-linear systems. Characteristics and stability of critical points of non-linear systems and stability analysis of such non-linear systems would be studied during the course. After successful completion of this course, a student would be able to understand the theory of linear and non-linear differential systems and will also be able to do practical problems related to solutions of linear systems and related to critical points and stability of solutions of non-linear systems.

#### MSM 206: PRACTICAL-II

This course aims to train the students for practical implementations of all the features of FORTRAN-90, 95 programming, which they study as a theory course MSM 203, i.e., Computer Programming. Also some problem solving techniques based on papers MSM 201 to MSM 205 will be taught.

## SEMESTER - III

## **MSM 301: FUNCTIONAL ANALYSIS**

The main objective of this course is to familiarize the students with normed linear spaces, Banach spaces, inner product spaces and Hilbert spaces. The four fundamental theorems: Hahn-Banach Theorem, Uniform Boundedness Theorem, Open Mapping Theorem and Closed Graph Theorem; related to continuous linear operators are the highlights of the course. We also make an excursion into Hilbert space, introducing basic concepts and proving the classical theorems associated with the names of Riesz, Bessel and Parseval. Self adjoint, unitary, normal, positive and projection operators also form a part of this course.

#### MSM 302: FLUID MECHANICS

Fluid mechanics is a branch of continuum mechanics which deals with mechanics of fluids (liquids and gases). Fluid mechanics can be divided into fluid statics, the study of fluids at rest and fluid dynamics which is concerned with flow of fluids and the forces acting on them. Fluid mechanics has a wide range of applications, including mechanical engineering, civil engineering, chemical engineering, geophysics, astrophysics, and biology. The first section of this course is introductory and the students will come to learn about basic concepts and equations including kinematics of fluid in motion, equation of continuity, general analysis of fluid motion, equation of motion and energy equations. The second section of the course is about the study of fluid dynamics of real or viscous fluids. A student will learn about law of viscosity, states of stress and rate of strain and relation between them, Navier-Stoke's equations of motion, vorticity equations, laminar flows problems and problems associated with steady flow in pipes.

## MSM 303: PRACTICAL-III (FORTRAN-90 FOR NUMERICAL METHODS)

Any mathematics treatment aims for a precise and analytical solution. However, there are many problems which are too long (or tedious) to solve by hand. Moreover, there are also many problems which simply do not have analytical solutions, or those whose exact solution is beyond our current state of knowledge. When such problems arise we can exploit numerical analysis to reduce the problem to one involving a finite number of unknowns and use a computer to solve the resulting equations. FORTRAN is the main language having capability to address problems of scientific computation involving numerical methods. A theory course in FORTRAN-90 programming as well as working knowledge of capable FORTRAN software are the essential requirement for this course.

## MSM 304: ADVANCED TOPOLOGY

The objective of this course is to familiarize the students with some advanced topics in topology. Starting from the convergence of sequences in topological spaces and in first axiom topological spaces we move on to the introduction and convergence of nets in topological spaces followed by canonical way of converting nets to filters and vice versa. The concepts of metrisable spaces and paracompactness also form a part of the course along with some topics from algebraic topology including the fundamental group, Euclidean simplexes, singular simplexes etc.

## MSM 305: ALGEBRIC CODING THEORY

The course contains systematic study of coding and communication of messages. This course is concerned with devising efficient encoding and decoding procedures using modern algebraic techniques. The course begins with basic results of error detection and error correction of codes, thereafter codes defined by generator and parity check matrices are given. The course also contains polynomial codes, Hamming codes, construction of finite fields and thereafter the construction of BCH codes. Linear codes, MDS codes, Reed-Solomon codes, Perfect codes, Hadamard matrices and Hadamard codes are also the part of the course.

## MSM 306: COMMUTATIVE ALGEBRA

The course is designed to give an exposer of the concepts in commutative rings and modules defined on commutative rings. The course contains exact sequences of modules, tensor product modules, localization, primary decomposition of an ideal. This course also contains Integrally closed domains, valuation rings, Noether's normalization Lemma, chain conditions on rings and modules, primary decomposition of an ideal in Notherian rings. Structure theorem of Artinian rings, Discrete valuation rings, Dedekind domains and fractional ideals of a domain are also the part of the course.

## **MSM 307: DIFFERENTIAL GEOMETRY**

Differential geometry is a mathematical discipline that uses the techniques of differential calculus, vector calculus and linear algebra to study problems in geometry. In differential geometry, we study the mathematical analysis of curves and surfaces in space. In this course, students will learn about curves in space and other related concepts like tangent, principal normal, curvature, binormal, torsion, curvature and locus of centre of curvature. Next, the concepts of surfaces, tangent plane, normal, envelopes, developable surfaces will be studied. Curves on a surface, including the topics of lines of curvature, conjugate systems, asymptotic lines, isometric lines, null lines, shall be taught. The students will also learn the equations of Gauss and of Codazzi and Geodesics and related curves.

## MSM 308: ELASTICITY

This course is in continuation to the course MSM 204 (Mechanics of Solids) being taught as a core paper in second semester. This paper deals with elastostatics problems on extension, torsion, bending and flexure of beams through the application of forces and couples. The techniques used to solve these problems involve the applications of complex analysis (analytic functions, conformal mappings) as well. The boundary value problems arising in plane elasticity are solved for analytical solutions. Some techniques of solving the three-dimensional elastodynamics problems are also discussed.

## MSM 309: FINANCIAL MATHEMATICS

No one can deny the fact that financial markets play a fundamental role in economic growth of nations by helping efficient allocation of investment of individuals to the most productive sectors of the economy. Financial sector has seen enormous growth over the past thirty years in the developed world. This growth has been led by the innovations in products referred to as financial derivatives that require great deal of mathematical sophistication and ingenuity in pricing and in creating an insurance or hedge against associated risks. Hence, this course is for anyone who is interested in the applications of finance, particularly advanced /latest business techniques. Students are required to know elementary calculus (derivatives and partial derivatives, finding maxima or minima of differentiable functions of one or more variables, Lagrange multipliers, the Taylor formula and integrals), probability (random variables and probability (binomial & normal) distributions, expectation, variance and covariance, conditional probability and independence) and linear algebra (systems of linear equations, add, multiply, transpose and invert matrices, and compute determinants).

## MSM 310: FUZZY SETS AND APPLICATIONS

Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling; and are facilitators for common-sense reasoning in decision making in the absence of complete and precise information. The last five decades have witnessed the significant progress made in the development of fuzzy sets and fuzzy logic theory and their application in all branches of science, engineering and socio-economic sciences. The main objective of this course is to familiarize the students with fuzzy sets, operations on fuzzy sets, fuzzy numbers, fuzzy relations, possibility theory and fuzzy logic.

## **MSM 311: INTEGRAL EQUATIONS**

This course is designed to get acquainted with the concept of integral equations and the methods to find their solutions. A student will learn about integral equations, their classifications, eigen values and eigen functions, method of successive approximations, iterative methods, resolvent kernel. Fredholm three theorems are main part of the first section. In the second section, symmetric kernels, Riesz-Fisher theorem, Hilbert-Schmidt, solution of a symmetric integral equation, Abel's integral equation and Cauchy type singular integral equation are learnt.

## MSM 312: MATHEMATICAL MODELING

A mathematical model is a description of a system (device or a phenomenon) using mathematical concepts and language. The process of developing a mathematical model is defined as mathematical modeling. A mathematical model may help to explain a system and to study the effects of different components, and to make predictions about the system. During this course, the students will learn basic concepts of mathematical modeling and to construct mathematical models for population dynamics, epidemic spreading, economics, medicine, arm-race, battle and genetics. Students will also learn Mathematical modeling through partial differential equations and probability generating function.

## MSM 313: MATHEMATICAL STATISTICS

Mathematical Statistics is very useful in all branches of science as well as all branches of social sciences. The concept of Mathematical Statistics is surely one of the popular branch of mathematics. The main aim of this course is to introduce Random distribution, Probability models, Mathematical Expectation, Correlation coefficient, Binomial, Poisson, Gamma, Chi-square, Normal, Bivariate normal distributions, t & F distributions, Stochastic convergence, Stochastic independence. An attempt has been made in this course to strike a balance between the different concepts of mathematical statistics.

## **MSM 315: NUMBER THEORY**

The concept of Number Theory is surely one of the oldest ideas of mathematics. The main aim of this course is to introduce Diophantine equations, Farey sequences, Geometry of numbers, continued fractions, partition functions. An attempt has been made in this course to strike a balance between the different concepts of number theory.

#### **OE 307: APPLIED NUMERICAL METHODS**

This course has been designed to introduce numerical methods to the students of the faculty of sciences. The students will come to learn different popular numerical methods for solving transcendental and polynomial equations, system of linear equations, eigen value problems, interpolation and curve fitting, numerical differentiation, numerical integration, solution of ordinary differential equations. After successful completion of the course, a student will be able to draw the algorithm of these numerical methods that will form the basis of implementation of numerical methods programs in either of programming languages.

## SEMESTER - IV

## MSM 401: MECHANICS AND CALCULUS OF VARIATIONS

Mechanics is an area of applied mathematics and physics that deals with motion of a body and the forces producing motion. This subject comprises of kinetics, statics, kinematics and dynamics but this course deals with dynamics of rigid bodies and of system of particles. The students will learn about moments and products of inertia, principal axes, kinetic energy of rigid bodies and general motion of a rigid body. Euler's dynamical equations and related two and three dimensional problems will also be taught during the course. The students will also learn about Lagrange's equations, variational principle, Hamiltonian and the theory of small oscillations of conservative holonomic dynamical systems.

Calculus of variations is an area that deals with maximizing or minimizing functionals. Functionals are often expressed as definite integrals involving functions and their derivatives. The interest is in extremal functions that make the functional to be stationary. In the portion of calculus of variation, which has many applications in mechanics and other fields of mathematics, the students will come to learn about Euler's equations, variational problems for functionals depending on one or more independent variables and/or on one or more dependent variables. Solution of some basic problems with their applications part will also be illustrated.

## **MSM 402: PARTIAL DIFFERENTIAL EQUATIONS**

The learning objective of this paper is to study the partial differential equations (PDE). A PDE is a differential equation that contains unknown multivariable functions and their partial derivatives. PDEs can be used to describe a wide variety of phenomena such as sound, heat, electrostatics, electrodynamics, fluid dynamics, elasticity and mechanics. During this course, a student will learn about partial differential equations including definition, classifications, analytical theory and methods of solutions of IVP, transport equations, Laplace's equation, Poisson's equation and heat equations. Derivation of Green's function and method of solving PDEs by Green's function approach will also be learnt. Other component of the learning objective is to study Wave equation, solutions of wave equation in different forms, Kirchhoff's and Poisson's formula, solution of non-homogeneous wave equation, solution of Laplace, heat and wave equations by method of separation of variables, similarity solutions and by using Fourier and Laplace transforms.

## MSM 403: PRACTICAL-IV (NUMERICAL METHODS USING MATLAB)

This practical paper is in continuation to the practical paper MSM-303, which deals with the use of numerical methods through FORTRAN-90 programming. In the present paper, all the techniques used in MSM-303 are used through MATLAB programming. In addition to that the plotting capabilities of MATLAB are used to verify the correctness and to analyse the properties of the solutions calculated. Superiority of MATLAB (an application software) over FORTRAN (as a programming language) is effectively realised through this laboratory course

## MSM 404: SEMINAR-II

In this course a student will learn to select the topic amongst syllabi of other courses prescribed in this semester. A student will learn to collect, review and to understand the literature and to present the contents of the topic so chosen. After the completion of this course the student will get an exposure towards self study and enhancement of presentation skills.

#### MSM 405: ADVANCED COMPLEX ANALYSIS

The objective of this course is to familiarize the students with some advanced topics in complex analysis. Starting from the compactness and convergence in the space of analytic functions we move on to establish the Runge's theorem and Mittag-Leffler's theorem followed by analytic continuation and

Riemann surfaces. Entire functions and the range of an analytic function are the concluding topics of this advance course in complex analysis.

## MSM 406: ADVANCED DISCRETE MATHEMATICS

The course consists of two sections. In the first section lattices are defined as algebraic structures. This section contains various types of lattices i.e. modular, distributive and complimented lattices. Chain conditions are defined and studied in modular lattices. The notion of independent elements in modular lattices is introduced. Boolean algebra has been introduced as an algebraic system. Basic properties of finite Boolean algebra and application of Boolean algebra to switching circuit theory is also given. Section two contains graph theory. In this section students will be taught connected graphs, Euler's theorem on connected graphs, trees and their basic properties. This section also contains fundamental circuits and fundamental cut-sets, planner graphs, vector space associated with a graph, and the matrices associated with graphs, paths, circuits and cut-sets. The contents of this paper find many applications in computer science and engineering science.

#### MSM 407: ADVANCED FUNCTIONAL ANALYSIS

Spectral theory is one of the main branches of modern functional analysis and its applications. The main objective of this course is to familiarize the students with some advanced topics in functional analysis which include spectral theory of linear operators in normed spaces, compact linear operators on normed spaces and their spectrum, and spectral theory of bounded self-adjoint linear operators. The theory of approximation is a very extensive field which has various applications. The fundamental ideas and aspects of approximation theory in normed and Hilbert spaces also form a part of this course.

#### MSM 408: ALGEBRAIC NUMBER THEORY

The concept of ALGEBRAIC NUMBER THEORY is surely one of the recent ideas of mathematics. The main aim of this course is to introduce algebraic numbers, algebraic integers, Liouville's Theorem Cyclotomic Polynomials Norm and trace, Ideals in the ring of algebraic number field, Dedekind domains, Fractional ideals, Chinese Remainder theorem, Different of an algebraic number field, Hurwitz constant, Ideal class group, Minkowski's bound and Quadratic reciprocity.

## MSM 409: ANALYTIC NUMBER THEORY

One objective of this course is to develop analytic number theory with the help of complex analysis. An attempt has been made in this course to make a balance between the different concepts of analytic number theory such as Arithmetical functions, prime number theorem, character group, Ramanujan's sum, Dirichlet characters and Gauss sums.

## MSM 410: BIO MATHEMATICS

This paper deals with a widely acceptable fact that many phenomena in ecology, biology and biochemistry can be modelled mathematically. The main emphasis is on mathematical modeling, with biology the sole application area. Biology offers a rich variety of topics that are amenable to mathematical modeling, but some of the genuinely interesting are touched in this paper. It is assumed that students have no knowledge of biology, but they are expected to learn a substantial amount during the course. The ability to model problems using mathematics may not require much of the memorization, but it does require a deep understanding of basic principles and a wide range of mathematical techniques. Students are required to know differential equations and linear algebra. Topics in stochastic modeling are also touched, which requires some knowledge of probability.

## MSM 411: BOUNDARY VALUE PROBLEMS

This course is based on the course mainly integral equations. The contents integral and boundary value problems and the techniques to find their analytical solution if possible.

Some problems pertaining to practical aspects have been included to the worthwhile applications of Boundary Value Problems. The mixed boundary value problems have also been included as an extension of Boundary Value Problems.

## **MSM 412: FLUID DYNAMICS**

Fluid dynamics is a sub-discipline of fluid mechanics that describes the flow of fluids (liquids and gases). It has several sub-disciplines, including aerodynamics (the study of air and other gases in motion) and hydrodynamics (the study of liquids in motion). Fluid dynamics has a wide range of applications in different fields of science and engineering including aerospace, aeronautical and petroleum engineering. The objective of the paper is to learn about two dimensional inviscid incompressible flows, sources, sinks, doublets, irrotational motion, complex velocity potential, two dimensional motion produced by the motion of a cylinder, kinetic energy of a rotating cylinder, theorem of Blasius and theorem of Kutta and Joukowski, axi-symmetric flow. A student will also learn about three-dimesional motion including motion of a sphere and liquid streaming past a fixed sphere, DÁlemberts's paradox, impulsive motion, vortex motion, motions due to circular and rectilinear vortices, dynamical similarity, Buckingham pi-theorem, Reynold's number, Prandtl's number and boundary layer theory.

## MSM 413: GENERAL MEASURE AND INTEGRATION THEORY

The main object of this course is to familiarize the students with the general theory of measure and integration, in particular, with measurable functions, sequences of measurable functions, integrable functions, product measures, finite signed measures and integration over locally compact spaces. Egoroff's theorem, Riesz-Weyl theorem, Radon-Nikodym theorem, Hahn decomposition, Jordan decomposition, Riesz-Markoff's theorem and Fubini's theorem are the highlights of this course.

## **MSM 414: LINEAR PROGRAMMING**

Real life systems can have dozens or hundreds of variables, or more, which may not be handled through standard algebraic techniques. Such systems are used every day in the organization and allocation of resources and are generally handled through linear programming based on "optimization techniques". Linear programming deals with the problems of maximizing or minimizing a linear function subject to linear constraints in the form of equalities or inequalities. The general process for solving linear-programming exercises is to graph the constraints to form a walled-off area called "feasibility region". Then, corners of this feasibility region are tested to find the highest (or lowest) value of the outcome (or resources).

## MSM 415: MATHEMATICAL ASPECTS OF SEISMOLOGY

Seismology is the study of earthquakes. An earthquake is a sudden release of energy inside the Earth due to fault or rupture there. This course has been designed to study applications of mathematics and theory of elasticity in the field of seismology. Students will learn how to model the generation and propagation of seismic energy mathematically.

Students will first learn about the interior of the Earth and basic concepts related to earthquakes viz. causes, observation and location of earthquakes, magnitude, energy, foreshocks and aftershocks etc. When an earthquake occurs, energy travels through the Earth. The students will next learn the mathematical representation of waves and wave characteristics, solutions of wave equation in different coordinate systems, plane and spherical waves. The phenomenon of dispersion and dispersive waves will also be taught in this course.

The Earth is not homogeneous, so the seismic waves face discontinuities as they propagate inside the Earth. The mathematical theory of elastic waves, their reflection and refraction will be studied in the section-II. Snell's law and partition of the energy at a free surface and at an interface will also be studied. Mathematical models for the propagation of surface waves, which cause most of the

destruction during an earthquake, will also be taught. Finally, mathematical models for the source problems will be studied and the students will learn to obtain analytical solutions of the Lamb's problems.

## **MSM 416: NON – COMMUTATIVE RINGS**

The course has been designed to give an exposer of the advanced ring theory. Course contains some special example of rings i.e. differential polynomial rings, group rings, skew group rings, triangular rings, Hurwitz's rings of integral quaternion's. Students are also taught DCC and ACC in triangular rings, Dedekind finite rings, simple and semi-simple modules, projective and injective modules. Nil radical and Jacobson radical of matrix rings are also part of the course. The course also contains subdirect product of rings and commutativity theorems of Jacobson-Herstein and Herstein-Kaplansky. Finally theory of finite division rings is given.

## MSM 417: WAVELET ANALYSIS

Wavelets meaning small waves are recent addition to scientific world. Methods dealing with properties of wavelets are called wavelet analysis. Wavelet analysis is a modern supplement to classical Fourier analysis. In some cases Wavelet analysis is much better than Fourier analysis in the sense that fewer terms suffice to approximate certain functions. The main objective of this course is to familiarize the students with the standard features of Fourier transforms along with more recent developments such as the discrete and fast Fourier transforms and wavelets. We consider the idea of a Multiresolution Analysis and the course we follow is to go from MRA to wavelet bases.

#### SYLLABUS FOR M.SC MATHEMATICS SEMESTER – I

MSM-101: ABSTRACT ALGEBRA

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section-I (Four Questions)**

Automorphisms and Inner automorphisms of a group G. The groups Aut(G) and Inn(G). Automorphism group of a cyclic group. Normalizer and Centralizer of a non-empty subset of a group G. Conjugate elements and conjugacy classes. Class equation of a finite group G and its applications. Derived group (or a commutator subgroup) of a group G. perfect groups. Simplicity of the Alternating group  $A_n$  ( $n \ge 5$ ). Zassenhaus's Lemma. Normal and Composition series of a group G. Scheier's refinement theorem. Jordan Holder theorem. Composition series of groups of order  $p^n$  and of finite Abelian groups. Caunchy theorem for finite groups. p-groups. Finite Abelian groups. Sylow p-subgroups. Sylow's Ist, IInd and IIIrd theorems. Application of Sylow theory to groups of smaller orders.

Commutators identities. Commutator subgroups. Three subgroups Lemma of P.Hall. Central series of a group G. Nilpotent groups. Centre of a nilpotent group. Subgroups and factor subgroups of nilpotent groups. Finite nilpotent groups. Upper and lower central series of a group G and their properties. Subgroups of finitely generated nilpotent groups. Sylow-subgroups of nilpotent groups. Solvable groups Derived series of a group G. Non-solvability of the symmetric group  $S_n$  and the Alternating group  $A_n$  ( $n \ge 5$ ). (Scope of the course as given in the book at Sr. No. 1).

## **Section-II (Four Questions)**

Modules, submodules and quotient modules. Module generated by a non-empty subset of an R-module. Finitely generated modules and cyclic modules. Idempotents. Homomorphism of R-modules. Fundamental theorem of homomorphism of R-modules. Direct sum of modules. Endomorphism rings  $\operatorname{End}_Z(M)$  and  $\operatorname{End}_R(M)$  of a left R-module M. Simple modules and completely reducible modules (semi-simple modules). Finitely generated free modules. Rank of a finitely generated free module. Submodules of free modules of finite rank over a PID. Representation of linear mappings and their ranks.

Endomorphism ring of a finite direct sum of modules. Finitely generated modules. Ascending and descending chains of sub modules of an R-module. Ascending and Descending change conditions (A.C.C. and D.C.C.). Noetherian modules and Noetherian rings. Finitely co-generated modules. Artinian modules and Artinian rings. Nilpotent elements of a ring R. Nil and nilpotent ideals. Hilbert Basis Theorem. Structure theorem for finite Boolean rings. Wedeerburn-Artin theorem and its consequences. Uniform modules. Primary modules. Nother-laskar Theorem. (Scope of the course as given in the book at Sr. No. 2).

#### **Recommended Books:**

I.S. Luthar and I.B.S. Passi : Algebra Vol. 1 Groups (Narosa publication House)

P.B. Bhattacharya S.R. Jain and S.R. Nagpal: Basic Abstract Algebra

**Reference Books:** 

1. I.D. Macdonald : Theory of Groups

2. Vivek Sahai and Vikas Bist : Algebra (Narosa publication House)

3. Surjit Singh and Quazi Zameeruddin : Modern Algebra (Vikas Publishing House 1990)

4.W.R. Scott : Group Theory

## MSM-102: COMPLEX ANALYSIS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section-I (Four Questions)**

Analytic functions, Harmonic functions, Uniquely Determined Analytic Functions, Reflection principle.

Elementary Functions: Exponential Functions, Logarithmic Functions, Trigonometric Functions, Hyperbolic Functions, Inverse Trigonometric Functions, Inverse Hyperbolic Functions, Complex exponents.

Complex Integration: Definite integral; Contours; Branch cuts; Cauchy-Goursat theorem. Simply connected domains, Multiply connected domains, Cauchy integral formula.

Morera's theorem; Liouville's theorem; Fundamental theorem of algebra; Maximum modulus principle;

Power series: Uniform and absolute convergence, differentiation and integration of power series, multiplication and division of power series; Taylor series; Laurentseries;

## **Section-II** (Four Questions)

Singularities; Poles; Residues.Cauchy's Residue Theorem; Zeros of an analytic function.

Evaluation of improper integrals; Jordan's lemma; Indentation; Integration along a branch cut; Definite integrals involving sines and cosines; winding number of closed curve; Open mapping theorem; Argument principle; Rouche's theorem; Schwarz Lemma

Transformations: linear, bilinear (Mobius), sine, z<sup>2</sup>, z<sup>1/2</sup>; Riemann surfaces;

Mapping: Isogonal; Conformal; Scale factors; Local inverses; Harmonic conjugates; Transformations of harmonic functions, Transformations of Boundary conditions;

## **Recommended Text Book:**

Churchill, R.V. and Brown, J.W., Complex Variables and Applications, Eighth edition; McGraw Hill International Edition, 2009.

#### **Reference Books:**

- 1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
- 2. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.
- 3. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
- 4. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
- 5. Mark J.Ablewitz and A.S.Fokas, Complex Variables: Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
- 6. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
- 7. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

# MSM-103: ORDINARY DIFFERENTIAL EQUATIONS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section –I (Four Questions)**

Preliminaries: Initial value problem and equivalent integral equation, ε-approximate solution, equicontinuous set of functions.Basic theorems: Ascoli- Arzela theorem, Cauchy –Peano existence theorem and its corollary. Gronwall's inequality.

Lipschitz condition. Uniqueness of solutions. Successive approximations. Picard-Lindelöf theorem. Continuation of solution, Maximal interval of existence, Extension theorem.

(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Higher order equations: Linear differential equation (LDE) of order n; Linear combinations, Linear dependence and linear independence of solutions. Wronskian theory: Definition, necessary and sufficient condition for linear dependence and linear independence of solutions of homogeneous LDE. Abel's Identity, Fundamental set.More Wronskian theory.Reduction of order.

Non-homogeneous LDE. Variation of parameters. Adjoint equations, Lagrange's Identity, Green's formula. Linear equation of order n with constant coefficients.

(Relevant portions from the books of 'Theory of Ordinary Differential Equations' by Coddington and Levinson and the book 'Differential Equations' by S.L. Ross)

## **Section-II(Four Questions)**

Linear second order equations: Preliminaries, self adjoint equation of second order, basic facts. Superposition Principle. Riccati's equation. Prüffer transformation. Zero of a solution. Abel's formula. Common zeros of solutions and their linear dependence.

Sturm theory: Sturm separation theorem, Sturm fundamental comparison theorem and their corollaries. Oscillatory and non-oscillatory equations. Elementary linear oscillations.

(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Second order boundary value problems(BVP): Linear problems; periodic boundary conditions, regular linear BVP, singular linear BVP; non-linear BVP. Sturm-Liouville BVP: definitions, eigen values and eigen functions. Orthogonality of functions, orthogonality of eigen functions corresponding to distinct eigen values.

Green's function. Applications of Green's function for solving boundary value problems. Use of Implicit function theorem and Fixed point theorems for periodic solutions of linear and non-linear equations.

(Relevant portions from the book 'Textbook of Ordinary Differential Equations' by Deo et all)

#### **Recommended books:**

- 1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill . 2000.
- 2. S.L. Ross, *Differential Equations*, John Wiley & Sons,
- 3. S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations*, Tata McGraw-Hill, 2006.

## **Reference books:**

1. P. Hartman, Ordinary Differential Equations, John Wiley & Sons NY, 1971.

- 2. G. Birkhoff and G.C. Rota, Ordinary Differential Equations, John Wiley & Sons, 1978.
- 3. G.F. Simmons, Differential Equations, Tata McGraw-Hill, 1993.
- 4. I.G. Petrovski, Ordinary Differential Equations, Prentice-Hall, 1966.
- 5. D. Somasundaram, Ordinary Differential Equations, A first Course, Narosa Pub., 2001.
- 6. Mohan C Joshi, *Ordinary Differential Equations, Modern Perspective*, Narosa Publishing House, 2006.

#### MSM-104: REAL ANALYSIS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section-I (Four Questions)**

Definition and existence of Riemann Stieltjes integral, properties of the integral, reduction of Riemann Stieltjes integral to ordinary Riemann integral, change of variable, integration and differentiation, the fundamental theorem of integral calculus, integration by parts, first and second mean value theorems for Riemann Stieltjes integrals, integration of vector-valued functions, rectifiable curves. (Scope as in Chapter 6 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

Sequences and series of functions: Pointwise and uniform convergence of sequences of functions, Cauchy criterion for uniform convergence, Dini's theorem, uniform convergence and continuity, uniform convergence and Riemann integration, uniform convergence and differentiation, convergence and uniform convergence of series of functions, Weierstrass M-test, integration and differentiation of series of functions, existence of a continuous nowhere differentiable function, the Weierstrass approximation theorem, the Arzela theorem on equicontinuous families. (Scope as in Chapter 9 (except 9.6) & Chapter 10 (except 10.3) of 'Methods of Real Analysis' by R.R. Goldberg).

## **Section-II** (Four Questions)

Functions of several variables: Linear transformations, the space of linear transformations on  $\mathbb{R}^n$  to  $\mathbb{R}^m$  as a metric space, open sets, continuity, derivative in an open subset of  $\mathbb{R}^n$ , chain rule, partial derivatives, directional derivatives, continuously differentiable mappings, necessary and sufficient conditions for a mapping to be continuously differentiable, contractions, the contraction principle (fixed point theorem), the inverse function theorem, the implicit function theorem. (Scope as in relevant portions of Chapter 9 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition)

Power Series: Uniqueness theorem for power series, Abel's and Tauber's theorem, Taylor's theorem, Exponential & Logarithmic functions, trigonometric functions, Fourier series, Gamma function (Scope as in relevant portions of Chapter 8 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

#### **Recommended Text:**

- 'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition) McGraw-Hill, 1976.
- 'Methods of Real Analysis' by R.R.Goldberg, Oxford and IHB Publishing Company, New Delhi, 1970 **Reference Books:**
- 1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
- 2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975.
- 3. A.J. White, Real Analysis; an introduction. Addison-Wesley Publishing Co., Inc., 1968.
- 4. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
- 5. Serge Lang, Analysis I & II, Addison-Wesley Publishing Company Inc., 1969.
- 6. S.C. Malik and Savita Arora, Mathematical Analysis, New Age International Limited, New Delhi,4<sup>th</sup> Edition 2010.
- 7.D. Somasundaram and B. Choudhary: A First Course in Mathematical Analysis, Narosa Publishing House, New Delhi, 1997

#### MSM-105: TOPOLOGY

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section-I (Four Questions)**

Definition and examples of topological spaces, Neighbourhoods, Neighbourhood system of a point and its properties, Interior point and interior of a set, interior as an operator and its properties, definition of a closed set as complement of an open set, limit point (accumulation point) of a set, derived set of a set, adherent point (Closure point) of a set, closure of a set, closure as an operator and its properties, boundary of a set, Dense sets. Base for a topology and its characterization, Base for Neighbourhood system, Sub-base for a topology. Relative (induced) Topology and subspace of a topological space. Alternate methods of defining a topology using 'properties' of 'Neighbourhood system', 'Interior Operator', 'Closed sets', Kuratowski closure operator. First countable, Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and a second countable space, Lindelof theorem.

Comparison of Topologies on a set, about intersection and union of topologies, the collection of all topologies on a set as a complete lattice Definition, examples and characterisations of continuous functions, composition of continuous functions, Open and closed functions, Homeomorphism, embedding. Tychonoff product topology, projection maps, their continuity and openness, Characterization of product topology as the smallest topology with projections continuous, continuity of a function from a space into a product of spaces. T<sub>0</sub>, T<sub>1</sub>, T<sub>2</sub>, Regular and T<sub>3</sub> Separation axioms, their characterization and basic properties i.e. hereditary and productive properties. Quotient topology w.r.t. a map, Continuity of function with domain a space having quotient topology, About Hausdorffness of quotient space

## **SECTION-II** (Four Questions)

Completely regular and Tychonoff (T  $_3$   $_{1/2}$ ), spaces, their hereditary and productive properties. Embedding lemma, Embedding theorem. Normal and T $_4$  spaces: Urysohn's Lemma, complete regularity of a regular normal space, Tietze's extension theorem (Statement only).

Definition and examples of filters on a set, Collection of all filters on a set as a p.o. set, finer filter, methods of generating filters/finer filters, Ultra filter (u.f.) and its characterizations, Ultra Filter Principle (UFP). Image of a filter under a function. Convergence of filters: Limit point (Cluster point) and limit of a filter and relationship between them, Continuity in terms of convergence of filters. Hausdorffness and filter convergence.

Compactness: Definition and examples of compact spaces, definition of a compact subset as a compact subspace, relation of open cover of a subset of a topological space in the sub-space with that in the main space, compactness in terms of finite intersection property (f.i.p.), continuity and compact sets, compactness and separation properties, closeness of continuous map from a compact space into a Hausdorff space and its consequence, Regularity and normality of a compact Hausdorff space. Compactness and filter convergence, Convergence of filters in a product space, compactness and product space. Tychonoff product theorem, Tychonoff space as a subspace of a compact Hausdorff space and its converse, compactification and Hausdorff compactification, Stone-Cech compactification, (Scope of the course is as given in chapters 1, 3, 4 & 5 of the Kelley's book given at Sr. No. 1).

## **Recommended Books:**

1. Kelley, J.L. : General Topology.

2. Munkres, J.R. : Topology, Second Edition, Prentice Hall of India/ Pearson.

#### MSM-106: PRACTICAL-I

External Theory Examination: 35 Marks

Internal Assessment: 15 Marks

Time: 4 Hours

Part A: Problem Solving

In this part problem solving techniques based on papers MSM 101 to MSM 105 will be taught.

## Part B: ANSI-C codes for following mathematical problems

- 1. Use of nested if...else in finding the smallest of four numbers.
- 2. To find if a given 4-digit year is a leap year or not.
- 3. To compute AM, GM and HM of three given real values.
- 4. To invert the order of digits in a given positive integral value.
- 5. Use series sum to compute sin(x) and cos(x) for given angle x in degrees. Then, check error in verifying  $sin^2x+cos^2(x)=1$ .
- 6. Verify  $\sum n^3 = \{\sum n \}^2$ , (where n=1,2,..,m) & check that prefix and postfix increment operator gives the same result.
- 7. Compute simple interest and compound interest for a given amount, time period, rate of interest and period of compounding.
- 8. Program to multiply two given matrices in a user defined function.
- 9. Calculate standard deviation for a set of values  $\{x(j), j=1,2,...,n\}$  having the corresponding frequencies  $\{f(j), j=1,2,...,n\}$ .
- 10. Write the user-defined function to compute GCD of two given values and use it to compute the LCM of three given integer values.
- 11. Compute GCD of 2 positive integer values using recursion / pointer to pointer.
- 12. Check a given square matrix for its positive definite form.
- 13. To find the inverse of a given non-singular square matrix.
- 14. To convert a decimal number to its binary representation.
- 15. Use array of pointers for alphabetic sorting of given list of English words.

## Note:

Every student will have to prepare a file to maintain practical record of the problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) two or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.

## SYLLABUS FOR M.SC MATHEMATICS SEMESTER – II MSM-201: ADVANCED ABSTRACT ALGEBRA

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **Section-I (Four Questions)**

Characteristic of a ring with unity. Prime fields Z/pZ and Q. Characterization of prime fields. Field extensions. Degree of an extension. Algebraic and transcendental elements. Simple field extensions. Minimal polynomial of an algebraic element. Conjugate elements. Algebraic extensions. Finitely generated algebraic extensions. Algebraic closure and algebraically closed fields. Splitting fields.

Finite fields. Flobineous automorphism of a finite field. Roots of unity, Cyclotomic polynomials and their irreducibility over Q. Normal extensions. Finite normal extensions as Splitting fields. Separable elements, separable polynomials and separable extensions. Theorem of primitive element. Perfect fields.

(Scope of the course as given in the book at Sr. No. 2).

## <u>Section – II (Four Questions)</u>

Galois extensions. Galois group of an extension. Dedekind lemma Fundamental theorem of Galois theory. Frobenius automorphism of a finite field. Klein's 4-group and Diheadral group. Galois groups of polynomials. Fundamental theorem of Algebra. Radicals extensions. Galois radical extensions. Cyclic extensions. Solvability of polynomials by radicals over Q. Symmetric functions and elementary symmetric functions. Construction with ruler and compass only.

(Scope of the course as given in the book at Sr. No. 2).

Similar linear transformations. Invariant subspaces of vector spaces. Reduction of a linear transformation to triangular form. Nilpotent transformations. Index of nilpotency of a nilpotent transformation. Cyclic subspace with respect to a nilpotent transformation. Uniqueness of the invariants of a nilpotent transformation. Primary decomposition theorem. Jordan blocks and Jordan canonical forms. Cyclic module relative to a linear transformation. Rational Canonicals form of a linear transformation and its elementary divisior. Uniqueness of the elementary divisior.

(Sections 6.4 to 6.7 of the book. Topics in Algebra by I.N. Herstein).

#### **Recommended Books:**

I.N. Herstein :Topics in Algebra (Wiley Eastern Ltd.)

P.B. Bhattacharya : Basic Abstract Algebra (Cambridge University Press 1995)

S.K. Jain & S.R. Nagpal

## **Reference Books:**

1. Vivek Sahai and Vikas Bist : Algebra (Narosa publication House)

2. Surjit Singh and Quazi Zameeruddin: Modern Algebra (Vikas Publishing House 1990)

3. Patrick Morandi : Field and Galois Theory (Springer 1996)

## **MSM-202: COMPUTER PROGRAMMING (THEORY)**

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## Section-I

Numerical constants and variables; arithmatic expressions; input/output statements; conditional flow; looping; logical expressions and control flow; functions; subroutines; arrays. (Relevant portions of chapters 1 to 10 of the recommended text book)

## **Section-II**

Format specifications; strings; array arguments; derived data types; processing files; pointers; FORTRAN 90 features; FORTRAN 95 features.

(Relevant portions of chapters 11-12, 14-15, 17-18 and 20-21 of the recommended text book)

## **Recommended Text**

V. Rajaraman: Computer Programming in FORTRAN 90 and 95; Printice-Hall of India Pvt. Ltd., New Delhi, 1997.

#### References

- 1. V. Rajaraman: Computer Programming in FORTRAN 77, Printice-Hall of India Pvt. Ltd., New Delhi, 1984.
- 2. J. F. Kerrigan: Migrating to FORTRAN 90, Orielly Associates, CA, USA, 1993.
- 3. M. Metcalf and J. Reid: FORTRAN 90/95 Explained, OUP, Oxford, UK, 1996.

## MSM-203: MEASURE AND INTEGRATION

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section-I (Four Questions)**

Lebesgue outer measure, elementary properties of outer measure, Measurable sets and their properties, Lebesgue measure of sets of real numbers, algebra of measurable sets, Borel sets and their measurability, characterization of measurable sets in terms of open, closed, F $\sigma$  and  $G_\delta$  sets, existence of a non-measurable set.

Lebesgue measurable functions and their properties, the almost everywhere concept , characteristic functions, simple functions, approximation of measurable functions by sequences of simple functions, Borel measurability of a function.

Littlewood's three principles, measurable functions as nearly continuous functions. Lusin's theorem, almost uniform convergence, Egoroff's theorem, convergence in measure, F.Riesz theorem that every sequence which is convergent in measure has an almost everywhere convergent subsequence.

The Lebesgue Integral: Shortcomings of Riemann integral, Lebesgue integral of a bounded function over a set of finite measure and its properties, Lebesgue integral as a generalization of the Riemann integral, Bounded convergence theorem, Lebesgue theorem regarding points of discontinuities of Riemann integrable functions.

## **Section-II** (Four Questions)

Integral of a non negative function, Fatou's lemma, Monotone convergence theorem, integration of series, the general Lebesgue integral, Lebesgue convergence theorem.

Differentiation and Integration: Differentiation of monotone functions, Vitali's covering lemma, the four Dini derivatives, Lebesgue differentiation theorem, functions of bounded variation and their representation as difference of monotone functions.

Differentiation of an integral, absolutely continuous functions, convex functions, Jensen's inequality.

The L<sup>p</sup> spaces, Minkowski and Holder inequalities, completeness of L<sup>p</sup> spaces, Bounded linear functionals on the L<sup>p</sup> spaces, Riesz representation theorem.

#### **Recommended Text:**

'Real Analysis' by H.L.Royden (3<sup>rd</sup> Edition) Prentice Hall of India, 1999.

## **Reference Books:**

- 1. G.de Barra, Measure theory and integration, Willey Eastern Ltd., 1981.
- 2. P.R.Halmos, Measure Theory, Van Nostrans, Princeton, 1950.
- 3. I.P.Natanson, Theory of functions of a real variable, Vol. I, Frederick Ungar Publishing Co., 1961.
- 4. R.G.Bartle, The elements of integration, John Wiley & Sons, Inc.New York, 1966.
- 5. K.R.Parthsarthy, Introduction to Probability and measure, Macmillan Company of India Ltd., Delhi, 1977.
- 6. P.K.Jain and V.P.Gupta, Lebesgue measure and integration, New age International (P) Ltd., Publishers, New Delhi, 1986.

## MSM-204: MECHANICS OF SOLIDS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION-I** (Four Questions)

Tensor Algebra: Coordinate-transformation, Cartesian Tensor of different order.

Properties of tensors. Isotropic tensors of different orders and relation between them. Symmetric and skew symmetric tensors. Tensor invariants. Deviatoric tensors. Eigen-values and eigen-vectors of a tensor.

Tensor Analysis: Scalar, vector, tensor functions, Comma notation, Gradient, divergence and curl of a vector / tensor field. (Relevant portions of Chapters 2 and 3 of book by D.S. Chandrasekharaiah and L.Debnath)

Analysis of Strain: Affine transformation, Infinitesimal affine deformation, Geometrical Interpretation of the components of strain. Strain quadric of Cauchy. Principal strains and invariance, General infinitesimal deformation. Saint-Venant's equations of compatibility.

Analysis of Stress: Stress Vector, Stress tensor, Equations of equilibrium, Transformation of coordinates. Stress quadric of Cauchy, Principal stress and invariants. Maximum normal and shear stresses. Mohr's circles. Examples of stress.

(Relevant portions of Chapter 1 & 2 of the book by I.S. Sokolnikoff).

## **SECTION-II** (Four Questions)

Equations of Elasticity: Generalised Hooks Law, Anisotropic symmetries, Homogeneous isotropic medium. Elasticity moduli for Isotropic media. Equilibrium and dynamic equations for an isotropic elastic solid. Strain energy function and its connection with Hooke's Law.

Beltrami-Michell compatibility equations. Uniqueness of solution. Clapeyron's theorem. Saint-Venant's principle.

(Relevant portions of Chapter 3 of book by I.S. Sokolnikoff).

Variational Methods: Theorem of minimum potential energy. Theorem of minimum complementary energy. Reciprocal theorem of Betti and Rayleigh. Ritz, Galerkin and Kantorovich methods. (Relevant portions of Chapter 7 of the book by I.S. Sokolnikoff).

#### **Recommended Books:**

I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata-McGraw Hill Publishing Company Ltd., New Delhi, 1977.

D.S. Chandrasekharaiah and L. Debnath, Continuum Mechanics, Academic Press, 1994.

## **Reference Books:**

- 1. A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity Dover Publications, New York.
- 2. Y.C. Fung. Foundations of Solid Mechanics, Prentice Hall, New Delhi, 1965.
- 3. Shanti Narayan, Text Book of Cartesian Tensor, S. Chand & Co., 1950.
- 4. S. Timeshenko and N. Goodier. Theory of Elasticity, McGraw Hill, New York, 1970.
- 5. I.H. Shames, Introduction to Solid Mechanics, Prentice Hall, New Delhi, 1975.

## MSM-205: SYSTEM OF DIFFERENTIAL EQUATIONS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## Section –I (Four Questions)

Linear differential systems: Definitions and notations. Linear homogeneous systems; Existence and uniqueness theorem, Fundamental matrix, Adjoint systems, reduction to smaller homogeneous systems.

Non-homogeneous linear systems; variation of constants.Linear systems with constant coefficients.Linear systems with periodic coefficients; Floquet theory.

(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

System of differential equations. Differential equation of order n and its equivalent system of differential equations. Existence theorem for solution of system of differential equations. Dependence of solutions on initial conditions and parameters: Preliminaries, continuity and differentiability.

(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

Maximal and Minimal solutions. Upper and Lower solutions. Differential inequalities. (Relevant portions from the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

## **Section –II (Four Questions)**

Autonomous systems: the phase plane, paths and critical points, types of critical points; Node, Center, Saddle point, Spiral point. Stability of critical points. Critical points and paths of linear systems: basic theorems and their applications. Critical points and paths of quasilinear systems.

(Relevant portions from the book 'Differential Equations' by S.L. Ross)

Stability Analysis: Asymptotic behaviour of linear system, generalized Gronwall's inequality. Formal approach of stability analysis. Phase portrait analysis.

(Relevant portions from the book of 'Ordinary Differential Equations' by Mohan C Joshi)

Stability of solution of system of equations with constant coefficients, linear equation with constant coefficients. Liapunov stability. Stability of quasi linear systems.

(Relevant portions from the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Limit cycles and periodic solutions: limit cycle, existence and non-existence of limit cycles, Benedixson's non-existence theorem. Half-path or Semiorbit, Limit set, Poincare-Benedixson theorem. (Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Theory of Ordinary Differential Equations' by Coddington and Levinson)

## **Recommended books:**

- 1. E.A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, Tata McGraw-Hill , 2000.
- 2. S.L. Ross, Differential Equations, John Wiley & Sons,
- 3. S.G. Deo, V. Lakshmikantham and V. Raghavendra, Textbook of Ordinary

- 4. Differential Equations, Tata McGraw-Hill, 2006.
- 5. Mohan C Joshi, Ordinary Differential Equations, Modern Perspective, Narosa Publishing House, 2006.

## **Reference books:**

- 1. P. Hartman, Ordinary Differential Equations, John Wiley & Sons NY, 1971.
- 2. G. Birkhoff and G.C. Rota, Ordinary Differential Equations, John Wiley & Sons, 1978.
- 3. G.F. Simmons, Differential Equations, Tata McGraw-Hill, 1993.
- 4. I.G. Petrovski, Ordinary Differential Equations, Prentice-Hall, 1966.
- 5. D. Somasundaram, Ordinary Differential Equations, A first Course, Narosa Pub., 2001.

#### MSM-206: PRACTICAL-II

External Theory Examination: 35 Marks

Internal Assessment: 15 Marks

Time: 4 Hours

Part A: Problem Solving

In this part problem solving techniques based on papers MSM 201 to MSM 205 will be taught.

## Part B: FORTRAN-90 codes for Mathematical problems

- 1. Given the centre and a point on the boundary of a circle, find its perimeter and area.
- 2. Calculate the area of a regular polygon for its given perimeter.
- 3. To check an equation  $ax^2 + by^2 + 2cx + 2dy + e = 0$  in (x, y) plane with given coefficients for representing parabola/ hyperbola/ ellipse/ circle or else.
- 4. To solve a quadratic equation with given coefficients, without using COMPLEX data type.
- 5. To find the location of a given point (x,y) i) at origin, ii) on x-axis or y-axis iii) in quadrant I, II, III or IV.
- 6. Use a function program for simple interest to display year-wise compound interest and amount, for given deposit, rate, time and compounding period.
- 7. To find the number of days in a given month of any given 4-digit year.
- 8. Use procedure for the greatest common divisor (gcd) of two given positive integers to compute the least common multiple (lcm) of three given positive integer values.
- 9. Find error in verifying  $\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$ , by approximating the  $\sin(x)$  and  $\cos(x)$  functions from the finite number of terms in their series expansions.
- 10. Use SELECT...CASE to calculate the income tax on a given income at the existing rates.
- 11. Use string operations to find if a given string is a palindrome or not.
- 12. To compute the arithmetic mean, geometric mean and harmonic mean for the values  $\{x(j), j=1,2,...,n\}$  having the corresponding frequencies  $\{f(j), j=1,2,...,n\}$ .
- 13. Product of two given matrices.
- 14. Least square fitting of a straight line to given set of points on a plane.
- 15. To solve a quadratic equation with given (complex-valued) coefficients, using COMPLEX data type.

## Note:

Every student will have to prepare a file to maintain practical record of the problems solved and the computer programs done during practical class-work. Examination will be conducted through a question paper set jointly by the external and internal examiners. An examinee will be asked to write the solutions in the answer book. An examinee will be asked to run (execute) two or more computer programs on a computer. Evaluation will be made on the basis of the examinee's performance in written solutions/programs, execution of computer programs and viva-voce examination.

## OPEN ELECTIVE PAPER OE-207: APPLIED ALGEBRA AND ANALYSIS

External Theory Examination: 35 Marks

Internal Assessment: 15 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section I**

Direct sum and Direct product (Kronecker product) of matrices and their properties, Gram matrices, Rank of a Gram matrix, Quadratic forms associated with a Gram matrix.

Lattices and their examples, distributive and complemented lattices, Boolean algebras, Boolean polynomials, Boolean functions, applications to switching circuit theory.

Lie algebras, examples of Lie algebras, subalgebras and ideals of Lie algebras, homorphisms of Lie algebras, derivation on Lie algebras, structure constants, quotient algebras, Lie algebras of dimensions 1 and 2, Heisenberg algebra.

(Scope of this section is as in relevant portions of the books mentioned at serial number 1, 2 and 3)

#### Section II

Metric spaces and normed linear spaces: Definitions and examples, metrics generated by a norm, Cauchy-Schwarz inequality, Holder inequality, Minkowski equality; co-ordinate spaces (Euclidean n-space, Unitary n-space), sequence spaces (Hilbert sequence space l<sub>2</sub>) and function spaces (C[a,b]).

Balls and boundedness: Balls and spheres in metric and normed linear spaces, relating norms and balls, symmetry and convexity of the unit sphere and unit balls, boundedness, diameter of a set, distance between sets.

Limit processes: Convergence of sequences, equivalence of convergence and co-ordinatewise convergence in Euclidean space, equivalent metrics and norms, Cauchy sequences, completeness, Banach spaces, completeness of discrete metric space, Euclidean n-spaces, Unitary n-spaces and B(X).

Application: Fixed points, fixed points of translation mapping, rotation mapping, reflection mapping; Contraction mapping, Banach's fixed point theorem, Applications of Banach's fixed point theorem in numerical analysis.

(Scope of this section is as in relevant portions of the book 'Introduction to the Analysis of Metric Spaces' by J. R. Giles)

#### **Recommended texts:**

- 1. C. L. Liu: Elements of Discrete Mathematics ( 2nd Edition ), Tata-McGraw-Hill Publishing Company Ltd., 2000.
- 2. Karin Erdmann and Mark J. Wildon: Introduction to Lie Algebras, Springer, 2006.
- 3. Shanti Narayan: A text book of matrices, S. Chand & Company (Pvt) Ltd., 1988.
- 4. J. R. Giles: Introduction to the Analysis of Metric Spaces, Cambridge University Press, 1987.

## SYLLABUS FOR M.SC.(MATHS) SEMESTER - III MSM 301: FUNCTIONAL ANALYSIS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **SECTION-I** (Four Questions)

Normed linear spaces, Banach spaces, finite dimensional normed spaces and subspaces, equivalent norms, compactness and finite dimension, F.Riesz's lemma. Bounded and continuous linear operators, differentiation operator, integral operator, bounded linear extension, linear functionals, bounded linear functionals, continuity and boundedness, normed spaces of operators, dual spaces with examples. (Scope as in relevant parts of Chapter 2 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

Hahn-Banach theorem for normed linear spaces, application to bounded linear functionals on C[a,b], Riesz-representation theorem for bounded linear functionals on C[a,b], adjoint operator, norm of the adjoint operator. Reflexive spaces, uniform boundedness theorem and some of its applications to the space of polynomials and fourier series. (Scope as in relevant parts of sections 4.1 to 4.7 of Chapter 4 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

## **SECTION-II** (Four Questions)

Strong and weak convergence, Open mapping theorem, bounded inverse theorem, closed linear operators, closed graph theorem, differential operator, relation between closedness and boundedness of a linear operator. (Scope as in relevant parts of sections 4.8, 4.12 and 4.13 of Chapter 4 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

Inner product spaces, Hilbert spaces and their examples, pythagorean theorem, Apolloniu's identity, Schwarz inequality, continuity of innerproduct, completion of an inner product space, subspace of a Hilbert space, orthogonal complements and direct sums, projection theorem, characterization of sets in Hilbert spaces whose space is dense. (Scope as in relevant parts of sections 3.1 to 3.3 of Chapter 3 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

Orthonormal sets and sequences, Bessel's inequality, series related to orthonormal sequences and sets, total(complete) orthonormal sets and sequences, Parseval's identity, separable Hilbert spaces. Representation of functionals on Hilbert spaces, Riesz representation theorem for bounded linear functionals on a Hilbert space, sesquilinear form, Riesz representation theorem for bounded sesquilinear forms on a Hilbert space. (Scope as in relevant parts of sections 3.4 to 3.6 and 3.8 of Chapter 3 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

Hilbert adjoint operator, its existence and uniqueness, properties of Hilbert adjoint operators, self adjoint, unitary, normal, positive and projection operators. (Scope is as in relevant parts of sections 3.9 to 3.10 of Chapter 3 and sections 9.3 to 9.6 of Chapter 9 of 'Introductory Functional Analysis with Applications' by E.Kreyszig.

## **Recommended Text Book:**

E.Kreyszig: Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1978.

## **Reference Books:**

1. G.F.Simmons: Introduction to Topology and Modern Analysis, McGraw Hill Book Co., New York, 1963.

- 2. C.Goffman and G.Pedrick: First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
- 3. G.Bachman and L.Narici, Functional Analysis, Academic Press, 1966.
- 4. L.A.Lustenik and V.J.Sobolev, Elements of Functional Analysis, Hindustan Publishing Corporation, New Delhi, 1971.
- 5. J.B.Conway: A Course in Functional Analysis, Springer-Verlag, 1990.
- 6. P.K.Jain, O.P.Ahuja and Khalil Ahmad: Functional Analysis, New Age International(P) Ltd. & Wiley Eastern Ltd., New Delhi, 1997.

## MSM 302: FLUID MECHANICS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION – I (Four Questions)**

Kinematics of fluid in motion: Real fluids and ideal fluids, Velocity at a point of a fluid. Lagrangian and Eulerian methods and the relation between them. Stream lines, path lines and streak liens, vorticity and circulation, Vortex lines, Acceleration and Material derivative.

Equation of continuity in vector form, Cartesian, cylindrical and spherical coordinates. Reynolds transport Theorem. Working rules for writing equation of continuity in some specific flows. General analysis of fluid motion. Properties of fluids- static and dynamic pressure. Boundary surfaces and boundary surface conditions. Inotational and rotational motions. Velocity potential.

Equation of Motion: Lagrange's and Euler's equations of Motion. Conservative field of force. Bernoulli's theorem. Applications of the Bernoulli Equation in one –dimensional flow problems. Kelvins circulation theorem, vorticity equation.

Energy equation for incompressible flow. Kinetic energy of irrotational flow. Kelvins minimum energy theorem. Mean potential over a spherical surface. Kinetic energy of infinite liquid. Uniqueness theorems. Elementary fluid motion in two dimensions: stream function, irrotational motion, complex potential, sources, sinks and doublets.

(Relevant portions from the recommended text books at Sr. No. 1 & 2)

## **SECTION – II (Four Questions)**

Real or Viscous fluids: Newton's Law of viscosity, Newtonian and non-Newtonian fluids, State of stress at a point, Nature of stresses, transformation of stress components. Nature of rate of strain, transformation of the rate of strain. Principal stress & strain rate. Relation between stress and rate of strain.

Navier – Stoke's equations of motion of a viscous fluid in vector from and in term of Cartesian, cylindrical and spherical coordinators. The energy equation. Diffusion of vorticity vorticity equation: Energy dissipation due to viscosity.

Laminar's flow: Steady flow between two parallel planes; plane Poiseuille flow, plane Couette flow, generalized plane Couette flow.

Steady flow in pipes; flow through a circular pipe (Hagen-Poiseuille flow), laminar steady flow between two coaxial circular cylinders, Laminar steady flow between concentric rotating cylinders. Uniqueness Theorem. Steady viscous flow in tubes of uniform elliptic, equilateral triangular and rectangular cross sections.

(Relevant portions from the recommended text books at Sr. No. 1 & 3)

## **Recommended Text Books:**

- 1. F. Chorlton, Text-book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.
- 2. Michael E.O. Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics, John Wiley & Sons, 1986.
- 3. S. W. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Ltd., New Delhi, 1976.

## **Reference Books:**

- 1. G.K. Batchelor, An Introduciton to Fluid Mechanics, Foundation Books, New Delhi, 1994.
- 2. A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics Springer-Verlag, New York, 1993.
- 3. L.D. Landau and E.M. Lipschitz, Fluid Mechanics Pergamon Press, London, 1985.
- 4. H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1979.
- 5. R.K. Rathy. An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi. 1976.9
- 6. A.D. Young, Boundary Layers, AIAA Education Series, Washington DC, 1989.
- 7. W.H. Besant and A.S. Ramsey, A Treatise on Hydromechanics, Part-II, CBS Publishers, Delhi, 1988.

### MSM 303: PRACTICAL-III

Max marks: 35+15 Time: 4 Hours

# **FORTRAN-90** for numerical methods

- 1. Solutions of simultaneous linear equations: Gauss-elimination method; Gauss-Jordan method; Jacobi method; Gauss-Seidel method;
- 2. Solution of algebraic / transcendental equations: bisection method; regula-falsi method; secant method; Newton-Raphson method; Muller method; Chevyshev method
- 3. Inversion of matrices: using adjoints; Jordan method
- 4. Interpolation: Lagrange interpolation; Newton interpolation; Hermite interpolation.
- 5. Numerical differentiation: methods based on i) Interpolation, ii) finite difference operators, iii) underdetermined coefficients
- 6. Numerical integration: Composite methods based on trapezoidal rule, Simpson1/3 rule and 3/8 rule; Romberg method
- 7. Solution of ordinary differential equations: Euler methods; Runge-Kutta methods; predictor-corrector methods;
- 8. Statistical problems on central tendency (mean, mode, median) and dispersion (standard variation, standard error)
- 9. Least square method to fit polynomial (curve) of given degree to given function (data set).

#### References:

Numerical methods for scientific and engineering computation
 MK Jain, SRK Iyengar and RK Jain; Wiley Eastern Ltd, 1984 (N Delhi).

### MSM 304: ADVANCED TOPOLOGY

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section-I (Four Questions)**

Convergence of sequences in topological spaces and in first axiom topological spaces, Nets in topological spaces, convergence of nets, Hausdorffness and convergence of nets, Subnets and cluster points, canonical way of converting nets to filters and vice versa, their convergence relations (Scope as in theorems 2-3,5-8 of Chapter 2 of Kelley's book at Ser. No.1)

Definition and examples of metrisable spaces, examples of non-metrisable spaces, metrisability of product of countably many metrisable spaces, metrisability of a subspace of a metrisable space, normality of a regular space which is second axiom or Lindelof, Urysohn's metrisation theorem.

Locally finite family, its equivalent forms, countably locally finite family, refinement, open refinement, closed refinement of a family, existence of countably locally finite open covering of a metrisable space, Nagata-Smirnov metrisation theorem, Paracompactness, normality of a paracompact Hausdorff space, paracompactness of a metrisable space and of regular Lindelof space, Smirnov metrisation theorem (Scope as in theorems 39.1-39.2. 40.3, 41.1-41.5 and 42.1 of Chapter 6 of the book by "Munkres" given at Sr. No. 2)

# **SECTION-II** (Four Questions)

Relation of homotopy of paths based at a point and homotopy classes, product of homotopy classes, Fundamental group, change of base point topological invariance of fundamental group. ( scope as in relevant parts of Chapter IV of the book by Wallace at Ser. No.3)

Euclidean simplex, its convexity and its relation with its faces, standard Euclidean simplex, linear mapping between Euclidean simplexes of same dimension (scope as in relevant parts of Chapter V of the book by Wallace at Ser. No.3)

Singular simplexes and group of p-chains on a space, special singular simplex on and its boundary, induced homomorphism between groups of chains, boundary of a singular simplex and a chain, cycles and boundaries on a space, homologous cycles, homology and relative homology groups, induced homomorphism on relative homology groups, induced homomorphism on relative homology groups, topological invariance of relative homology groups, Prisms, homotopic maps and homology groups. (scope as in relevant parts of Chapter VI of the book by Wallace at Ser. No.3)

Join of a point and a chain, Barycentric subdivision operator B, diameter of a Euclidean simplex and a singular simplex, operator H and its relation with B, representation of an element of a relative cycle made up of singular simplexes into members of a given open cover of the space, the excision theorem (scope as in relevant parts of Chapter VII of the book by Wallace at Ser. No.3)

### **Books recommended:**

1. Kelley, J.L. : General Topology.

2. Munkres, J.R. : Topology, Second Edition, Prentice Hall of India/ Pearson.

3. Wallace, A.H. : Introduction to Algebraic Topology Vol. I

- 1. K. Chandrasekhara Rao, Topology, Narosa Publishing House Delhi, 2009.
- 2. Fred H. Croom, Principles of Topology, Cengage Learning, 2009.
- 3. K.D. Joshi, Introduction to General Topology, Wiley Eastern Ltd, 2006.
- 4. C.W.Patty, Foundation of Topology, Jones & Bertlett, 2009.
- 5. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Book Company, 1963.

### MSM 305: ALGEBRAIC CODING THEORY

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION – I (Four Questions)**

Block Codes. Minimum distance of a code. Decoding principle of maximum likelihood. Binary error detecting and error correcting codes. Group codes. Minimum distance of a group code (m, m+1) parity check code. Double and triple repition codes. Matrix codes. Generator and parity check matrices. Dual codes. Polynomial codes. Exponent of a polynomial over the binary field. Binary representation of a number. Hamming codes. Minimum distance of a Hamming code.

Finite fields. Construction of finite fields. Primitive element of a finite field. Irreducibility of polynomials over finite fields. Irreducible polynomials over finite fields. Primitive polynomials over finite fields. Automorphism group of  $GF(q^n)$ . Normal basis of  $GF(q^n)$ . The number of irreducible polynomials over a finite field. The order of an irreducible polynomial. Generator polynomial of a Bose-Chaudhuri-Hocqhenghem codes (BCH codes) construction of BCH codes over finite fields. (Chapter 1, 2, 3 and 4 of the book given at Sr. No. 1 and Section 7.1 to 7.3 of the book given at Sr. No.

(Chapter 1, 2, 3 and 4 of the book given at Sr. No. 1 and Section 7.1 to 7.3 of the book given at Sr. No. 2).

# **SECTION – II (Four Questions)**

Linear codes. Generator matrices of linear codes. Equivalent codes and permutation matrices. Relation between generator and parity-check matrix of a linear codes over a finite field. Dual code of a linear code. Self dual codes. Weight distribution of a linear code. Weight enumerator of a linear code. Hadamard transform. Macwilliams identity for binary linear codes.

Maximum distance separable codes. (MDS codes). Examples of MDS codes. Characterization of MDS codes in terms of generator and parity check matrices. Dual code of a MDS code. Trivial MDS codes. Weight distribution of a MDS code. Number of code words of minimum distance d in a MDS code. Reed solomon codes.

Hadamard matrices. Existence of a Hadamard matrix of order n. Hadamard codes from Hadamard matrices Cyclic codes. Generator polynomial of a cyclic code. Check polynomial of a cyclic code. Equivalent code and dual code of a cyclic code. Idempotent generator of a cyclic code. Hamming and BCH codes as cyclic codes. Perfect codes. The Gilbert-varsha-move and Plotkin bounds. Self dual binary cyclic codes.

(Chapter 5, 6, 9 & 11 of the book at Sr. No. 1).

#### **Recommended Text Books:**

1. L.R. Vermani: Elements of Algebraic Coding Theory (Chapman and Hall Mathematics)

2. Steven Roman: Coding and Information Theory (Springer Verlag)

# MSM 306: COMMUTATIVE ALGEBRA

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION-I** (Four Questions)

Zero divisors, nilpotent elements and units, Prime ideals and maximal ideals, Nil radical and Jacobson radical, Comaximal ideals, Chineese remainder theorem, Ideal quotients and annihilator ideals. Extension and contraction of ideals. Exact sequences. Tensor product of module Restriction and extension of scalars. Exactness property of the tensor product. Tensor products of algebras.

Rings and modules of sections. Localization at the prime ideal P. Properties of the localization. Extended and contracted ideals in rings of fractions.

Primary ideals, Primary decomposition of an ideal, Isolated prime ideals, Multiplicatively closed subsets.

# **SECTION-II (Four Questions)**

Integral elements, Integral closure and integrally closed domains, Going-up theorem and the Going-down theorem, valuation rings and local rings, Noether's normalization lemma and weak form of nullstellensatz Chain condition, Noetherian and Artinian modules, composition series and chain conditions.

Noetherian rings and primary decomposition in Noetherian rings, radical of an ideal. Nil radical of an Artinian ring, Structure Theorem for Artinian rings, Discrete valuation rings, Dedekind domains, Fractional ideals.

(Scope of the course is as given in Chapter 1 to 9 of the recommended text).

# **Recommended Text Book:**

M.F.Atiyah, FRS and I.G.Macdonald Introduction to Commutative Algebra (Addison-Wesley Publishing Company)

#### **Reference Books:**

1. N.S.Gopal Krishnan, Oxonian Press Pvt. Ltd. Commutative Algebra

2. Zariski, Van Nostrand Princeton(1958) Commutative Algebra(Vol. I)

### MSM 307: DIFFERENTIAL GEOMETRY

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **Section –I (Four Questions)**

Curves: Tangent, principal normal, curvature, binormal, torsion, Serret-Frenet formulae, locus of center of curvature, spherical curvature, locus of centre of spherical curvature, curve determined by its intrinsic equations, helices, spherical indicatrix of tangent, etc., involutes, evolutes, Bertrand curves.

Envelopes and Developable Surface: Surfaces, tangent plane, normal. One parameter family of surfaces; Envelope, characteristics, edge of regression, developable surfaces. Developables associated with a curve; Osculating developable, polar developable, rectifying developable. Two parameter family of surfaces; Envelope, characteristic points and examples.

(Relevant portions from the books 'Differential Geometry of Three Dimensions' by C.E. Weatherburn)

# **Section-II(Four Questions)**

Curvilinear Coordinates, First order magnitudes, directions on a surface, the normal, second order magnitudes, derivatives of **n**, curvature of normal section, Meunier's theorem.

Curves on a surface: Principal directions and curvatures, first and second curvatures, Euler's theorem, Dupin's indicatrix, the surface z = f(x, y), surface of revolution. Conjugate systems; conjugate directions, conjugate systems. Asymptotic lines, curvature and torsion. Isometric lines; isometric parameters. Null lines, minimal curves.

The equations of Gauss and of Codazzi: Gauss's formulae for  $r_{11}$ ,  $r_{12}$ ,  $r_{22}$ , Gauss characteristic equation, Mainardi-Codazzi relations, alternative expression, Bonnet's theorem, derivatives of the angle  $\omega$ .

Geodesics: Geodesic property, equations of geodesics, surface of revolution, torsion of a geodesic. Curves in relation to Geodesics; Bonnet's theorem, Joachimsthal's theorems, vector curvature, geodesic curvature, Bonnet's formula.

(Relevant portions from the books 'Differential Geometry of Three Dimensions' by C.E. Weatherburn)

#### **Recommended Book:**

C.E. Weatherburn, Differential Geometry of Three Dimensions, Radha Publishing House, Calcutta.

- 1. J.A. Thorpe, Introduction to Differential Geometry, Springer-verlag.
- 2. B.O. Neill, Elementary Differential Geometry, Academic Press, 1966.
- 3. S. Sternberg, Lectures on Differential Geometry, Prentice- Hall, 1964.
- 4. R.S. Millman and G.D. Parker, Elements of Differential Geometry, Prentice-Hall, 1977.
- 5. W. Kleingenberg, A course in Differential Geometry, Springer-verlag.

### **MSM 308: ELASTICITY**

**External Theory Examination: 70 Marks** 

**Internal Assessment: 30 Marks** 

Time: 3 Hours

Note: Examiner is expected to set four questions from each section. Examinees are required to attempt five questions but at least two questions from each section.

# **Section-I** (four questions)

Extension: Extension of beams, bending of beams by own weight and terminal couples,;

Torsion: Torsion of cylindrical bars; Torsional rigidity. Torsion and stress functions. Lines of shearing stress. Simple problems related to equilateral triangle, grooves; Torsion of rectangular beam, Torsion of triangular prism, Torsion problems through conformal mapping; Torsion-membrane analogy; Torsion of hollow beams, Torsion of anisotropic beams; Flexure/bending of circular/elliptic /rectangular beams; Deformation /torsion of cylinders by lateral loads;

[sections 30 – 38, 44-47, 51-57, 63-64 (Chapter 4) of Sokolnikoff (1977)]

# **Section –II (four questions)**

Two dimensional problems: Plane stress. Generalized plane stress. Airy stress function. General solution of biharmonic equation, Stresses and displacements in terms of complex potentials. The structure of functions of  $\phi(z)$  and  $\psi(z)$ . First and second boundary-value problems in plane elasticity. Existence and uniqueness of the solutions. Basic problem for circular finite/infinite region.

Three dimensional problems: general solutions; concentrated forces; deformation by normal loads; The problem of Boussinesq; Elastic sphere: pressures, harmonics, equilibrium; Integration method; Thermoelastic problems; Vibrations in elastic solids.

[sections 65-74, 77-78 (Chapter 5) & sections 90-102 (Chapter 6) of Sokolnikoff (1977)]

- 1. I.S. Sokolnikoff, Mathematical Theory of Elasticity, Tata McGraw Hill Publishing Company Ltd., New Delhi, 1977.
- 2. A.E.H. Love, A Treatise on the Mathematical Theory of Elasticity Dover Publications, New York.
- 3. Y.C. Fung. Foundations of Solid Mechanics, Prentice Hall, New Delhi, 1965.
- 4. D.S. Chandrasekharaiah and L. Debnath, Continuum Mechanics, Academic Press, 1994.
- 5. S. Timeshenki and N. Goodier. Theory of Elasticity, McGraw Hill, New York, 1970.
- 6. I.H. Shames, Introduction to Solid Mechanics, Prentice Hall, New Delhi, 1975.

### MSM 309: FINANCIAL MATHEMATICS

# **External Theory Examination: 70 Marks**

Internal Assessment: 30 Marks
Time: 3 Hours

Note: Examiner is expected to set four questions from each section. Examinees are required to attempt five questions but at least two questions from each section.

# **Section-I** (four questions)

Fundamentals of Financial Mathematics Asset Price Model Black-Scholes Analysis Variations on Black-Scholes models

# **Section – II (four questions)**

Numerical Methods American Option Exotic Options Path-Dependent Options Bonds and Interest Rate Derivatives Stochastic calculus

- 1. Financial Mathematics: I-Liang Chern Department of Mathematics, National Taiwan University
- 2. Sheldon M. Ross, An Introduction to Mathematical Finance, Cambridge Univ. Press.
- 3. Robert J. Elliott and P. Ekkehard Kopp. Mathematics of Financial Markets, Springer-Verlag, New York Inc.
- 4. Robert C. Marton, Continuous-Time Finance, Basil Blackwell Inc.
- 5. Daykin C.D., Pentikainen T. and Pesonen M., Practical Risk Theory for Actuaries, Chapman & Hall.

### MSM 310: FUZZY SETS AND APPLICATIONS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all taking four questions from each section. The candidate is required to attempt five questions selecting at least two questions from each section.

## **SECTION-I** (Four Questions)

Fuzzy Sets: Basic definitions,  $\alpha$ -cuts, strong  $\alpha$ -cuts, level set of a fuzzy set, support of a fuzzy set, the core and height of a fuzzy set, normal and subnormal fuzzy sets, convex fuzzy sets, cutworthy property, strong cutworthy property, standard fuzzy set operations, standard complement, equilibrium points, standard intersection, standard union, fuzzy set inclusion, scalar cardinality of a fuzzy set, the degree of subsethood (Scope as in relevant parts of sections 1.3-1.4 of Chapter 1 of the book by Klir & Yuan ).

Additional properties of  $\alpha$ -cuts involving the standard fuzzy set operators and the standard fuzzy set inclusion, Representation of fuzzy sets, three basic decomposition theorems of fuzzy sets Extension principle for fuzzy sets: the Zedah's extension principle, Images and inverse images of fuzzy sets, proof of the fact that the extension principle is strong cutworthy but not cutworthy (Scope as in relevant parts of Chapter 2 of the book by Klir & Yuan )

Operations on fuzzy sets: types of operations, fuzzy complements, equilibrium of a fuzzy complement, equilibrium of a continuous fuzzy complement, first and second characterization theorems of fuzzy complements (Scope as in relevant parts of sections 3.1 and 3.2 of Chapter 3 of the book by Klir & Yuan).

Fuzzy intersections (t-norms), standard fuzzy intersection as the only idempotent t-norm, standard intersection, algebraic product, bounded difference and drastic intersection as examples of t-norms, decreasing generator, the Pseudo-inverse of a decreasing generator, increasing generators and their Pseudo-inverses, convertion of decreasing generators and increasing generators to each other, characterization theorem of t-norms(statement only). Fuzzy unions (t-conorms), standard union, algebraic sum, bounded sum and drastic union as examples of t-conorms, characterization theorem of t-conorms (Statement only), combination of operations, aggregation operations (Scope as in relevant parts of sections 3.3 to 3.6 of Chapter 3 of the book by Klir & Yuan).

### **SECTION-II (Four Questions)**

Fuzzy numbers, relation between fuzzy number and a convex fuzzy set, characterization of fuzzy numbers in terms of its membership functions as piecewise defined functions, fuzzy cardinality of a fuzzy set using fuzzy numbers, arithmetic operations on fuzzy numbers, extension of standard arithmetic operations on real numbers to fuzzy numbers, lattice of fuzzy numbers, (R, MIN, MAX) as a distributive lattice, fuzzy equations, equation A+X=B, equation A.X=B (Scope as in relevant parts of Chapter 4 of the book by Klir & Yuan)

Fuzzy Relations: Crisp and fuzzy relations, projections and cylindrical extensions, binary fuzzy relations, domain, range and height of a fuzzy relation, membership matrices, sagittal diagram, inverse of a fuzzy relation, composition of fuzzy relations, standard composition, max-min composition, relational join, binary relations on a single set, directed graphs, reflexive irreflexive, antireflexive, symmetric, asymmetric, artisymmetric, transitive (max-min transitive), non transitive, antitransitive fuzzy relations. Fuzzy equivalence relations, fuzzy compatibility relations,  $\alpha$ -compatibles, complete  $\alpha$ -cover, reflexive undirected graphs, fuzzy ordering relations, fuzzy upper bound, fuzzy pre ordering, fuzzy weak ordering, fuzzy strict ordering, fuzzy morphisms. Supicompositions of Fuzzy relations, Inf-i compositions of Fuzzy relations.

(Scope as in the relevant parts of Chapter 5 of the book by Klir & Yuan)

Possibility Theory: Fuzzy measures, continuity from below and above, semicontinuous fuzzy measures, examples and simple properties; Evidence Theory, belief measure, superadditivity, monotonicity, plausibility measure, subadditivity, basic assignment, its relation with belief measure and plausibility measure, focal element of basic assignment, body of evidence, total ignorance, Dempster's rule of combination, examples; Possibility Theory, necessity measure, possibility measure, implications, possibility distribution function, lattice of possibility distributions, joint possibility distribution. Fuzzy sets and possibility theory, Possibility theory versus probability theory (Scope as in the relevant parts of Chapter 7 of the book by Klir & Yuan)

Fuzzy Logic: An overview of classical logic, about logic functions of two variables, Multivalued logics, Fuzzy propositions, Fuzzy Quantifiers, Linguistic Hedges, Inference from conditional fuzzy propositions, inference from conditional and qualified propositions, inference from unqualified propositions. (Scope as in the relevant parts of Chapter 8 of the book by Klir & Yuan)

# **Recommended Book:**

G. J. Klir and B. Yuan: Fuzzy Sets and Fuzzy: Logic Theory and Applications, Prentice Hall of India, 2008

- 1. Kwang H. Lee, First Course on Fuzzy Theory and Applications, Springer International Edition, 2005.
- 2. H.J. Zimmerman, Fuzzy Set Theory and its Applications, Allied Publishers Ltd., New Delhi, 1991.
- 3. John Yen, Reza Langari, Fuzzy Logic Intelligence, Control and Information, Pearson Education, 1999.
- 4. A.K. Bhargava, Fuzzy Set Theory, Fuzzy Logic & their Applications, S. Chand & Company Pvt. Ltd., 2013.

# **MSM 311: INTEGRAL EQUATIONS**

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **SECTION-I (Four Questions)**

Definition of Integral Equations and their classifications. Eigen values and Eigen functions. Special kinds of Kernel Convolution Integral. The inner or scalar product of two functions. Reduction to a system of algebraic equations. Fredholm alternative, Fredholm theorem, Fredholm alternative theorem, An approximate method.

Method of successive approximations, Iterative scheme for Fredholm and Volterrra Integral equations of the second kind. Conditions of uniform convergence and uniqueness of series solution. Some results about the resolvent Kernel. Application of iterative scheme to Volterra integral equations of the second kind.

Classical Fredholm's theory, the method of solution of Fredholm equation, Fredholm's First theorem, Fredholm's second theorem, Fredholm's third theorem.

(Relevant portions from the Chapters 1, 2, 3 & 4 of the book "Linear Integral Equations, Theory & Techniques by R.P.Kanwal").

# **SECTION-II (Four Questions)**

Symmetric Kernels, Introduction, Complex Hilbert space. An orthonormal system of functions, Riesz-Fisher theorem, A complete two-Dimensional orthonormal set over the rectangle  $a \le s \le b, c \le t \le d$ . Fundamental properties of Eigenvalues and Eigenfunctions for symmetric Kernels. Expansion in eigen functions and Bilinear form. Hilbert-Schmidt theorem and some immediate consequences.

Definite Kernels and Mercer's theorem. Solution of a symmetric Integral Equation. Approximation of a general  $\ell_2$ -Kernel(Not necessarily symmetric) by a separable Kernel. The operator method in the theory of integral equations. Rayleigh-Ritz method for finding the first eigenvalue.

The Abel Intergral Equation. Inversion formula for singular integral equation with Kernel of the type h(s)-h(t),  $0<\alpha<1$ , Cauchy's principal value for integrals solution of the Cauchy-type singular integral equation, closed contour, unclosed contours and the Riemann-Hilbert problem. The Hilbert-Kernel, solution of the Hilbert-Type singular Intergal equation.

(Relevant portions from the Chapter 7 & 8 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

- 1. R.P.Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, New York.
- 2. S.G.Mikhlin, Linear Integral Equations (translated from Russian) Hindustan Book Agency, 1960.
- 3. I.N.Sneddon, Mixed Boundary Value Problems in potential theory, North Holland, 1966.
- 4. I, Stakgold, Boundary Value Problems of Mathematical Physics Vol.I, II, Mac.Millan, 1969.
- 5. Pundir and Pundir, Integral Equations and Boundary value problems, Pragati Prakashan, Meerut.

MSM 312: MATHEMATICAL MODELING

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

Note: Examiner is expected to set four questions from each section. Examinees are required to attempt

five questions but at least two questions from each section.

Section-I (Four Questions)

The process of Applied Mathematics; mathematical modeling: need, techniques, classification and illustrative;

mathematical modeling through ordinary differential equation of first order; qualitative solutions through

sketching.

Mathematical modeling in population dynamics, epidemic spreading and compartment models; mathematical

modeling through systems of ordinary differential equations; mathematical modeling in economics, medicine,

arm-race, battle.

Section-II (Four questions)

Mathematical modeling through ordinary differential equations of second order. Higher order (linear) models.

Mathematical modeling through difference equations: Need, basic theory; mathematical modeling in probability

theory, economics, finance, population dynamics and genetics.

Mathematical modeling through partial differential equations: simple models, mass-balance equations, variational

principles, probability generating function, traffic flow problems, initial & boundary conditions.

**References:** 

J.N. Kapur: Mathematical Modeling, Wiley Eastern Ltd., 1990 (Relevant portions; Chapters 1 to 6.)

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### MSM 313: MATHEMATICAL STATISTICS

**External Theory Examination: 70 Marks** 

**Internal Assessment: 30 Marks** 

**Time: 3 Hours** 

Note: Examiner is expected to set four questions from each section. Examinees are required to attempt five questions but at least two questions from each section.

# **Section-I** (four questions)

Random distribution: preliminaries, Probability density function, Probability models, Mathematical Expectation, Chebyshev's Inequality; Conditional probability, Marginal and conditional distributions, Correlation coefficient, Stochastic independence.

Frequency distributions: Binomial, Poissson, Gamma, Chi-square, Normal, Bivariate normal distributions.

Distributions of functions: Sampling, Transformations of variables: discrete and continuous; t & F distributions; Change of variable technique; Distribution of order; Moment-generating function technique; other distributions and expectations.

# **Section-II** (four questions)

Limiting distributions: Stochastic convergence, Moment generating function, Related theorems.

Intervals: Random intervals, Confidence intervals for mean, differences of means and variance; Bayesian estimation.

Estimation & sufficiency: Point estimation, sufficient statistics, Rao-Blackwell Theorem, Completeness, Uniqueness, Exponential PDF, Functions of parameters; Stochastic independence.

- **1.** Introduction to Mathematical Statistics (RV Hogg, AT Craig) Amerind Pub. Co. Pvt. Ltd. New Delhi, 1972. (Chapters 1 to 7)
- 2. Fundamentals of Mathematical Statistics (SC Gupta, VK Kapoor) Sultan Chand & Sons (2007)

### **MSM 315: NUMBER THEORY**

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **SECTION – I (Four Questions)**

The equation ax+by = c, simultaneous linear equations, Pythagorean triangles, assorted examples, ternary quadratic forms, rational points on curves.

Farey sequences, rational approximations, Hurwitz theorem, irrational numbers, Geometry of Numbers, Blichfeldt's principle, Minkowski's Convex body theorem Lagrange's four square theorem.

# **SECTION – II (Four Questions)**

Euclidean algorithm, infinite continued fractions, irrational numbers, approximations to irrational numbers, Best possible approximations, Periodic continued fractions, Pell's equation.

Partitions, Ferrers Graphs, Formal power series, generating functions and Euler's identity, Euler's formula, bounds on P(n), Jacobi's formula, a divisibility property.

# **Recommended Text Book:**

An Introduction to the Theory of Numbers 
Ivan Niven

Herbert S. Zuckerman Hugh L: Montgomery John Wiley & Sons(Asia)Pte.Ltd. (Fifth Edition)

### **OE 307: APPLIED NUMERICAL METHODS**

External Theory Examination: 35 Marks

Internal Assessment: 15 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **SECTION-I**

**Solution of Polynomial and Transcendental Equations**: Bisection method, iteration methods based on first degree ( secant method, Regula-Falsi method, Newton-Raphson method) and second degree equations (Muller method and Chebyshev method), methods for complex roots.

**Solution of Systems of Linear Equations:** Direct Methods; Forward substitution and back substitution method, Gauss elimination method, Gauss-Jordan method, Triangularization method, Cholesky Method. Iterative methods; Gauss-Seidel method, successive over relaxation method. Eigen values and eigen vectors; Jacobi's method, Given's method, Rutishauser method, power method.

**Interpolation and Approximation:** Finite difference operators, interpolating polynomials using finite differences; Gregory-Newton forward difference interpolation, Stirling and Bessel interpolations. Piecewise and spline interpolation, cubic spline interpolation. Least squares approximation.

# **SECTION-II**

**Numerical Differentiation**: Methods based on interpolation, methods based on finite differences, methods based on undetermined coefficients, extrapolation methods.

**Numerical Integration**: Newton-Cotes method, trapezoidal rule, Simpson's rule, Gauss-Legendre integration method, Gauss-Chebyshev integration method. Composite Trapezoidal and Simpson's Rule, Romberg Integration. Double integration.

**Solution of Differential Equations**: Initial value problem; Euler's method, Taylor series method, Runge-Kutta Methods, System of Differential Equations. Multistep methods; Adams-Bashforth methods, Milne-Simpson method, predictor-corrector method.

Boundary Value Problems: Linear second order differential equations, non-linear second order differential equations.

#### **Recommended Text Book:**

Jain, M. K., Iyengar, S.R.K.| and Jain, R.K., Numerical Methods for Scientific and Engineering Computation, 6th Edition, New Age International Publishers, 2012.

- 1. Mathews, John H. and Fink Kurtis D., Numerical Methods Using Matlab, Fourth edition; PHI Learning Private Ltd., 2009.
- 1. Gourdin, A. and Boumahrat, M., Applied Numerical Methods, PHI Learning Private Ltd., 1996.

# SYLLABUS FOR <u>M.SC.(MATHS)</u> SEMESTER - IV MSM 401: MECHANICS AND CALCULUS OF VARIATIONS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **SECTION – I (Four Questions)**

Moments and products of inertia, the theorems of parallel and perpendicular axes. Angular momentum of a rigid body about a fixed point and about fixed axes, principal axes.

Kinetic energy of a rigid body rotating about a fixed point. Momental ellipsoid – equimomental system, coplanar distributions. General motion of a rigid body.

Problems illustrating the law of motion, problems illustrating the law of conservation of angular momentum, problems illustrating the law of conservation of energy, problems illustrating impulsive motion.

Euler's dynamical equations for the motion of a rigid body about a fixed point, further properties of rigid motion under no forces. Some problems on general three-dimensional rigid body motion, the rotating earth.

(Relevant portions from the book 'Textbook of Dynamics' by F. Chorlton).

# **SECTION – II (Four Questions)**

Functional and its variation, Euler's (Euler-Lagrange) equations, variational problems for functionals depending on one independent and one dependent variable(s) and its (i) first derivative (ii) higher derivatives with fixed end conditions, Variational problems for functionals of the form  $\int_a^b F(x,y_1(x),y_2(x),\ldots,y_n(x),y_1'(x),y_2'(x),\ldots,y_n'(x))dx, \int_a^b F(x,y(x),y'(x),y'(x),y''(x),\ldots,y_n'(x))dx.$ 

Functionals dependent on functions of several independent variables. Variational problems in parametric form. Natural boundary conditions and transition conditions, Invariance of Euler's equation. Conditional extremum. Variational problem with moving boundaries. Some basic problems in calculus of variations: shortest distance, minimum surface of revolution, Brachistochrone problem, isoperimetric problem and geodesic problems.

(Relevant portions from the text books recommended at Sr. No. 3 & 4).

Note on dynamical systems, preliminary notions, generalized coordinates and velocities. Virtual work and generalized forces, Derivation of Lagrange's equations for a holonomic system, case of conservative forces. Generalized components of momentum and impulse. Lagrange's equations for impulsive forces. Kinetic energy as a quadratic function of velocities. Equilibrium configurations for conservative holonomic dynamical systems. Theory of small oscillations of conservative holonomic dynamical systems.

(Relevant portions from the book 'Textbook of Dynamics' by F. Chorlton).

Variational principles in Mechanics: Hamilton's principle, the principle of least action, Lagrange's equations for potential forces. Hamiltonian and canonical equations of Hamilton. Basic integral invariant of Mechanics. Canonical transformations, Hamilton Jacobi equation.

(Relevant portions from the text book recommended at Sr. No. 2).

# **Recommended Text Books:**

- 1. F. Chorlton: Text book of dynamics, CBS, 1985.
- 2. F. Gantmacher, Lectures in Analytic Mechanics, Khosla Publishing House, New Delhi.
- 3. Francis B. Hilderbrand, Methods of Applied Mathematics, Prentice Hall,
- 4. A.S. Gupta, Calculus of variations with Aplications, Printice-Hall of India Pvt. Ltd., New Delhi, 1999.

- 1. H. Goldstein, Classical Mechanics (2nd edition), Narosa Publishing House, New Delhi.
- 2. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.
- 3. S.K. Sinha, Classical Mechanics, Narosa, 2009.
- 4. Louis N. Hand and Janet D. Finch, Analytical Mechanics, Cambridge University Press, 1998.

# MSM 402: PARTIAL DIFFERENTIAL EQUATIONS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

### **SECTION – I (Four Questions)**

Partial Differential Equations (PDE) of k<sup>th</sup> order: Definition, examples and classifications. Initial value problems. Transport equations homogeneous and non-homogeneous, Radial solution of Laplace's Equation: Fundamental solutions, harmonic functions and their properties, Mean value Formula.

Poisson's equation and its solution, strong maximum principle, uniqueness, local estimates for harmonic functions, Liouville's theorem, Harnack's inequality.

Green's function and its derivation, representation formula using Green's function, symmetry of Green's function, Green's function for a half space and for a unit ball. Energy methods: uniqueness, Drichlet's principle.

Heat Equations: Physical interpretation, fundamental solution. Integral of fundamental solution, solution of initial value problem, Duhamel's principle, non-homogeneous heat equation, Mean value formula for heat equation, strong maximum principle and uniqueness. Energy methods. (Relevant portions from the recommended text books given at Sr. No. 1 & 2)

# **SECTION – II (Four Questions)**

Wave equation- Physical interpretation, solution for one dimentional wave equation, D'Alemberts formula and its applications, Reflection method, Solution by spherical means Euler-Poisson\_Darboux equation. Kirchhoff's and Poisson's formula (for n=2, 3 only).

Solution of non –homogeneous wave equation for n=1,3. Energy method. Uniqueness of solution, finite propagation speed of wave equation.

Non-linear first order PDE- complete integrals, envelopes, Characteristics of (i) linear, (ii) quasilinear, (iii) fully non-linear first order partial differential equations. Hamilton Jacobi equations.

Other ways to represent solutions: Method of Separation of variables for the Hamilton Jacobi equations, Laplace, heat and wave equations. Similarity solutions (Plane and traveling waves, solitones, similarity under Scaling).

Fourier Transform, Laplace Transform, Convertible non linear into linear PDE, Cole-Hop Transform, Potential functions, Hodograph and Legendre transforms. Lagrange and Charpit methods. (Relevant portions from the recommended text books given at Sr. No. 1 & 2)

#### **Recommended Text Books:**

- 1. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, American Mathematical Society, 2014.
- 2. I.N. Snedden, Elements of Partial Differential Equations, International Edition, McGraw-Hill, Singapore, 1986.
- 3. John F. Partial Differential Equations, Springer-Verlag, New York, 1971.

- 1. T. Amarnath, An Elementary Course in Partial Differential Equations, Jones & Bartlett Publishers, 2009
- 2. P. Parsad and R. Ravindran, Partial Differential Equations, New Age / International Publishers, 2005.

#### MSM 403: PRACTICAL-IV

Time: 4 hrs. Max marks: 35+15

# **Numerical methods using MATLAB**

- 1. Solutions of simultaneous linear equations: Gauss-elimination method; Gauss-Jordan method; Jacobi method; Gauss-Seidel method;
- 2. Solution of algebraic / transcendental equations: bisection method; regula-falsi method; secant method; Newton-Raphson method; Muller method; Chevyshev method
- 3. Inversion of matrices: Adjoint matrix method; Jordan method;
- 4. Interpolation: Lagrange interpolation; Newton interpolation; Hermite interpolation.
- 5. Numerical differentiation: methods based on i) Interpolation, ii) finite difference operators, iii) undetermined coefficients;
- 6. Numerical integration: Composite methods based on trapezoidal rule, Simpson1/3 rule and 3/8 rule; Romberg method
- 7. Solution of ordinary differential equations: Euler methods; Runge-Kutta methods; predictor-corrector methods;
- 8. Statistical problems on central tendency (mean, mode, median) and dispersion (standard variation, standard error);
- 9. Least square method to fit polynomial (curve) of given degree to given function (data set);
- 10. Plotting of special functions.

- 1. Numerical methods for scientific and engineering computation, (MK Jain, SRK Iyengar, RK Jain), Wiley Eastern ltd, N. Delhi (1984).
- 2. MATLAB Primer, Seventh Edition, (Timothy A. Davis, Kermit Sigmon), CHAPMAN & HALL/CRC

### MSM 405: ADVANCED COMPLEX ANALYSIS

External Theory Marks: 70 Internal Assessment Marks: 30

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all taking four questions from each section. The candidate is required to attempt five questions selecting at least two questions from each section.

# **Section-I (Four Questions)**

Convex functions and Hadamard's three circles theorem, The Phragmen-Lindelof Theorem

The space of continuous functions  $C(G,\Omega)$ , Arzela-Ascoli theorem, Spaces of analytic functions, Hurwitz's theorem, Montel's theorem.

Spaces of meromorphic functions, Riemann mapping theorem, infinite products, Weierstrass factorization theorem, Factorization of sine function, Gamma function and its properties, functional equation for gamma function, Bohr-Mollerup theorem, Reimann-zeta function, Riemann's functional equation, Euler's theorem.

Runge's theorem, Mittag-Leffler's theorem. Analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation along a curve, Power series method of analytic continuation, Schwarz reflection principle. Monodromy theorem and its consequences.

## **Section –II (Four Questions)**

Basic properties of harmonic functions, Harmonic function as a disk, Poisson's Kernel. Dirichlet problem for a unit disk, Harnack's inequality, Harnack's theorem. Dirichlet problem for a region, Green's function.

Entire functions: Jensen's formula, Poisson – Jensen formula. The genus and order of an entire function, Hadamard's factorization theorem.

The range of an analytic function: Bloch's theorem, Little-Picard theorem, Schottky's theorem, Montel-Carathedory theorem, Great Picard theorem.

#### **Recommended Text Book:**

Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

- 1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
- 2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
- 3. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
- 4. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
- 5. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
- 6. Mark J.Ablewitz and A.S.Fokas, Complex Variables: Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
- 7. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
- 8. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

### MSM 406: ADVANCED DISCRETE MATHEMATICS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION-I** (Four Questions)

Partially ordered sets and lattices. Lattice as an algebraic system. Sublattices. Isomorphism of lattices. Distributive and modular lattices. Lattices as intervals. Similar and projective intervals. Chains in lattices. Zassenhaus's Lemma and Schreier Theorem, Composition chain and Jordan Holder Theorem. Chain conditions. Fundamental dimensionality relation for modular lattices. Decomposition theory for lattices with ascending chain conditions, i.e. reducible and irreducible elements. Independent elements in lattices.

Points (atoms) of a lattice. Complemented lattices. Chain conditions and complemented lattices. Boolean algebras. Conversion of a Boolean algebra into a Boolean ring with unity and vice-versa. Direct product of Boolean algebras. Uniqueness of finite Boolean algebras. Boolean functions and Boolean expressions. Application of Boolean algebra to switching circuit theory.

(Chapter 7 of the book given at Sr. No. 2 & relevant portions of the chapter 12 of the book given at Sr. No. 3).

# **SECTION-II (Four Questions)**

Graphs, Konisberg seven bridges problem. Finite and infinite graphs. Incidence vertex. Degree of a vertex. Isolated and pendant vertices. Null graphs. Isomosphism of graphs. Subgraphs, walks, paths and circuits. Connected and disconnected graphs. Components of a graph. Euler graphs. Hamiltonian paths and circuits. The traveling salesman problem. Trees and their properties. Pendant vertices in a tree. Rooted and binary tree. Spanning tree and fundamental circuits. Spanning tree in a weighted graph.

Cutsets and their properties. Fundamental circuits and cutsets. Connectivity and separability. Network flows. Planner graphs. Kuratowski's two graphs. Representation of planner graphs. Euler formula for planner graphs. Vector space associated with a graph. Basis vectors of a graph. Circuit and cutset subspaces. Intersection and joins of  $W_C$  and  $W_S$ . Incidence matrix A(G) of a graph G, Submatrices of A(G), Circuit matrix, Fundamental circuit matrix, and its rank, Cutset matrix, path matrix and adjacency matrix of a graph.

(Chapter 1,2,3 & 4, Theorems 5.1 to 5.6 of chapter 5, chapter 6 & 7 of the book given at Sr. No. 1). (Relevant portion of the book given at Sr. No. 3)

# **Recommended Text Books:**

1.	Narsingh Deo	Graph Theory with application to Engineering and
		Computer Science, Prentice Hall of India.
2.	Nathan Jacobson	Lectures in Abstract Algebra Vol.I, D.Van Nostrand Company, Inc.
3.	L.R. Vermani and	A course in discrete Mathematical structures(Imperial College
	Shalini	Press London 2011)

### MSM 407: ADVANCED FUNCTIONAL ANALYSIS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all taking four questions from each section. The candidate is required to attempt five questions selecting at least two questions from each section.

## **SECTION-I** (Four Questions)

Spectrum of a bounded operator: point spectrum, continuous spectrum and residual spectral properties of bounded linear operators. Approximate point spectrum and compression spectrum, spectral mapping theorem for polynomials. Holomorphy of R $_{\lambda}$ , spectral radius, Banach algebras, properties of Banach algebras (Scope as in relevant parts of Chapter 7 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

Compact linear operators on normed spaces, compactness criterion, conditions under which the limit of a sequence of compact linear operators is compact, weak convergence and compact operators, adjoint of compact operators, Spectral properties of compact linear operators, eigen values of compact linear operators, cosedness of the range of  $T_{\lambda}$ , Operator equations involving compact linear operators, further theorems of Fredholm type. Fredholm alternative. (Scope as in relevant parts of Chapter 8 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

# **SECTION-II (Four Questions)**

Spectral theory of bounded self-adjoint linear operators: spectral properties of bounded self adjoint operators, positive operators and their properties, spectral representation of a self adjoint compact operator, spectral family of a bounded self adjoint operator and its properties, spectral representation of bounded self adjoint linear operators, theorem for bounded self adjoint linear operators, properties of p(T), extension of spectral theorem to continuous functions, properties of the spectral family of a bounded self adjoint operator. (Scope as in relevant parts of Chapter 9 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

Approximation in normed spaces, existence and uniqueness theorem for best approximations, strict convexity, uniform approximation, extremal point, Harr condition, Harr uniqueness theorem for best approximation, Chebyshev Polynomials, Approximation in Hilbert space, splines (Scope as in relevant parts of Chapter 6 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)

### **Recommended Text Book:**

E.Kreyszig: Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1978.

- 1. Simmons, G.F.: Introduction to Topology and Modern Analysis (Section 6.4-6.8), Mc Graw-Hill (1963) International Book Company
- 2. R. Bhatia, Notes on Functional Analysis, TRIM series, Hindustan Book Agency, India, 2009.
- 3. J.E. Conway, A course in Operator Theory, Graduate Studies in Mathematics, Volume 21, AMS (1999)
- 4. Martin Schechter, Principles of Functional Analysis, American Mathematical Society, (2004)
- 5. W. Rudin, Functional Analysis, TMH Edition, 1974

### MSM 408: ALGEBRAIC NUMBER THEORY

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION – I (Four Questions)**

Algebraic numbers and algebraic integers. Transcendental Numbers. Liouville's Theorem for real Algebraic numbers. Thue Theorem and Roth's theorem (statement only). Algebraic numberfield K. Theorem of Primitive elements. Liouville's Theorem for complex algebraic numbers. Minimal polynomial of an algebraic integer. Primitive m-th roots of unity. Cyclotomic Polynomials. Norm and trace of algebraic numbers and algebraic integers. Bilinear form on algebraic number field K.

Integral basis and discriminant of an algebraic number field. Index of an element of K. Ring  $O_k$  of algebraic integers of an algebraic number field K. Ideals in the ring of algebraic number field K. Integrally closed domains. Dedekind domains. Fractional ideals of K. Factorization of ideals as a product of prime ideals in the ring of algebraic integers of an algebraic number field K. G.C.D. and L.C.M. of ideals in  $O_k$ . Chinese Remainder theorem.

# **SECTION – II (Four Questions)**

Different of an algebraic number field K. Dedekind theorem. Euclidean rings. Hurwitz Lemma and Hurwitz constant. Equivalent fractional ideals. Ideal class group. Finiteness of the ideal class group. Class number of the algebraic number field K. Diophantine equations Minkowski's bound.

Quadratic reciprocity Legendre Symbol. Gauss sums. Law of quadratic reciprocity. Quadratic fields. Primes in special progression.

# **Recommended Text Book:**

Jody Esmonde and M.Ram Murty Problems in Algebraic Number Theory

(Springer Verlag, 1998)

**Reference Books:** 

1. Paulo Ribenboim Algebraic Numbers

2. R. Narasimhan Algebraic Number Theory and S. Raghavan Mathematical Pamphlets-4.

Tata Institute of Fundamental Research (1966).

MSM 409: Analytic Number Theory

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

# **SECTION – I (Four Questions)**

Arithmetical functions, Mobius function, Euler totient function, relation connecting Mobius function and Euler totient function, Product formula for Euler totient function, Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius inversion formula, Mangoldt function, multiplicative functions, Multiplicative functions and Dirichlet multiplication. Inverse of completely multiplicative function, Liouville's function, divisor function, generalized convolutions, Formal power-series, Bell series of an arithmetical function, Bell series and Dirichlet multiplication, Derivatives of arithmetical functions, Selberg identity. Asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas, average order of divisor functions, average order of Euler totient function.

Application to the distribution of lattice points visible from the origin, average order of Mobius function and Mangoldt function, Partial sums of a Dirichlet Product, applications to Mobius function and Mangoldt function, Legendre's identity, another identity for the partial sums of a Dirichlet product. Chebyshev's functions, Abel's identity, some equivalent forms of the prime number theorem. Inequalities for  $\pi(n)$  and  $P_n$ . Shapiro's Tauberian theorem. Applications of Shapiro's theorem. An

asymptotic formula for the partial sums  $\sum_{p \le x} \left( \frac{1}{p} \right)$ . Partial sums of the Mobius function. Brief sketch of an elementary proof of the prime number theorem; Selberg's asymptotic formula.

## **SECTION – II (Four Questions)**

Elementary properties of groups, construction of subgroups, characters of finite abelian groups, the character group, orthogonality relations for characters, Dirichlet characters, Sums-involving Dirichlet characters, Nonvanishing of  $L(1,\chi)$  for real nonprincipal  $\chi$ .

Dirichlet's theorem for primes of the form 4n-1 and 4n+1. Dirichlet's theorem. Functions periodic modulo K, Existence of finite Fourier series for periodic arithmetical functions. Ramanujan's sum and generalizations, multiplicative properties of the sums  $S_k$  (n). Gauss sums associated with Dirichlet characters. Dirichlet characters with nonvanishing Gauss sums. Induced moduli and primitive characters, properties of induced moduli conductor of a character. Primitive characters and separable Gauss sums. Finite fourier series of the Dirichlet characters. Polya's inequality for the partial sums of primitive characters.

## **Recommended Book:**

Tom M. Apostol Introduction to Analytic Number Theory

### MSM 410: BIO-MATHEMATICS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

Note: Examiner is expected to set four questions from each section. Examinees are required to attempt five questions but at least two questions from each section.

# **Section-I (Four Questions)**

Population Dynamics: The Malthusian growth; The Logistic equation; A model of species competition; The Lotka-Volterra predator-prey model

Age-structured Populations: Fibonacci's rabbits;

The golden ratio  $\Phi$ ; The Fibonacci numbers in a sunflower; Rabbits are an age-structured population; Discrete age-structured populations; Continuous age-structured populations; The brood size of a hermaphroditic worm;

Stochastic Population Growth: A stochastic model of population growth; Asymptotics of large initial populations; Derivation of the deterministic model; Derivation of the normal probability distribution; Simulation of population growth.

## **Section-II** (four questions)

Infectious Disease Modeling: The SI model; The SIS model; The SIR epidemic disease model; Vaccination; The SIR endemic disease model; Evolution of virulence.

Population Genetics: Haploid genetics; Spread of a favored allele; Mutation-selection balance; Diploid genetics; Sexual reproduction; Spread of a favored allele; Mutation-selection balance; Heterosis; Frequency-dependent selection; Linkage equilibrium; Random genetic drift.

Biochemical Reactions: The law of mass action; Enzyme kinetics; Competitive inhibition; Allosteric inhibition; Cooperativity.

Sequence Alignment: DNA; Brute force alignment; Dynamic programming; Gaps; Local alignments; Software.

- 1. Mathematical Biology, Lecture notes for MATH 4333, (Jeffrey R. Chasnov)
- 2. Mathematical Biology I. An Introduction, Third Edition, (J.D. Murray)

### MSM 411: BOUNDARY VALUE PROBLEMS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION – I (Four Questions)**

Applications to Ordinary Differential Equations; Initial value problems, Boundary Value Problems. Dirac Delta functions. Green's function approach to reduce boundary value problems of a self-adjoint-differential equation with homogeneous boundary conditions to integral equation forms. Green's function for N<sup>th</sup>-order ordinary differential equation. Modified Green's function.

Applications to partial differential equations: Integral representation formulas for the solution of the Laplace and Poisson Equations. The Newtonian, single-layer and double-layer potentials, Interior and Exterior Dirichlet problems, Interior and Exterior Neumann problems. Green's function for Laplace's equation in a free space as well as in a space bounded by a ground vessel. Integral equation formulation of boundary value prolems for Laplace's equation. Poisson's Integral formula. Green's function for the space bounded by grounded two parallel plates or an infinite circular cylinder. The Helmholtz equation.

(Relevant portions from the Chapter 5 & 6 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

# **SECTION-II (Four Questions)**

Integral Transform methods: Introduction, Fourier transform. Laplace transform. Convolution Integral. Application to Volterra Integral Equations with convolution-type Kernels. Hilbert transform. Applications to mixed Boundary Value Problems: Two-part Boundary Value problems, Three-part-Boundary Value Problems, Generalized Three-part Boundary Value problems.

Integral equation perturbation methods: Basic procedure, Applications to Electrostatics, Low-Reynolds-Number Hydrodynamics: Steady stokes Flow, Boundary effects on Stokes flow, Longitudnal oscillations of solids in stokes Flow, Steady Rotary Stokes Flow, Rotary Oscillations in Stokes Flow, Rotary Oscillation in Stokes Flow, Oseen Flow-Translation Motion, Oseen Flow-Rotary motion Elasticity, Boundary effects, Rotation, Torsion and Rotary Oscillation problems in elasticity, crack problems in elasticity, Theory of Diffraction.

( Relevant portions from the Chapter 9, 10 & 11 of the book "Linear Integral Equation, Theory and Techniques by R.P.Kanwal").

- 1. R.P.Kanwal, Linear Integral Equations, Theory and Techniques, Academic Press, New York.
- 2. S.G.Mikhlin, Linear Integral Equations (translated from Russian) Hindustan Book Agency, 1960.
- 3. I.N.Sneddon, Mixed Boundary Value Problems in potential theory, North Holland, 1966.
- 4. I, Stakgold, Boundary Value Problems of Mathematical Physics Vol.I, II, Mac.Millan, 1969.
- 5. Pundir and Pundir, Integral equations and Boundary value problems, Pragati Prakashan, Meerut.

#### MSM 412: FLUID DYNAMICS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION – I (Four Questions)**

Two-dimensional inviscid incompressible flows: Stream function, Irrotational motion in two and three dimensions. Image system of a source, sink and doublet.

Complex velocity potential. Thomson-circle theorem. Two- dimensional irrotational motion produced by motion of circular cylinder.

Two dimensional motion produced by the motion of cylinder of arbitrary uniform cross section in an infinite mass of liquid at rest at infinity. Motion due to elliptic cylinder in an infinite mass of liquid. Kinetic energy of liquid contained in rotating elliptic cylinder, circulation about elliptic cylinder.

Theorem of Blasius. Theorem of Kutta and Joukowski. Kinetic energy of a cyclic and acyclic irrotational motion. Axis-symmetric flows, Stoke's stream function, Stoke's stream functions of some basic flows.

(Relevant portions from the recommended text books given at Sr. No. 1 to 4)

# **SECTION – II (Four Questions)**

Three–dimensional motion: Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere. Equation of motion a sphere. D'Alembert's paradox, impulsive motion, initial motion of liquid contained in the intervening space between two concentric spheres.

Vortex motion and its elementary properties. Kelvin's proof of permanence. Motions due to circular and rectilinear vortices. Infinite rowes of line vortices.

(Relevant portions from the recommended text books given at Sr. No. 1, 2 & 3)

Dynamical similarity. Buckingham pi-theorem, Reynolds number. Prandtl's boundary layer, boundary layer equations in two dimensions. Blasius solution Boundary layer thickness. Displacement thickness, Karman integral conditions, separation of boundary layer.

(Relevant portions from the recommended text books given at Sr. No. 1 to 4)

### **Recommended Text Books:**

- 1. W.H. Besant and A.S. Ramsey, A Treatise on Hydromechanics, Part-II, CBS Publishers, Delhi, 1988.
- 2. F. Chorlton, Text-book of Fluid Dynamics, C.B.S. Publishers, Delhi, 1985.
- 3. Michael E.O. Neill and F. Chorlton, Ideal and Incompressible Fluid Dynamics, John Wiley & Sons, 1986.
- 4. S. W. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Ltd., New Delhi, 1976.event

- 1. G.K. Batchelor, An Introduciton to Fluid Mechanics, Foundation Books, New Delhi, 1994.
- 2. A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics Springer-Verlag, New York, 1993.
- 3. L.D. Landau and E.M. Lipschitz, Fluid Mechanics Pergamon Press, London, 1985.
- 4. H. Schlichting, Boundary Layer Theory, McGraw Hill Book Company, New York, 1979.

- 5. R.K. Rathy. An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi. 1976.
- 6. A.D. Young, Boundary Layers, AIAA Education Series, Washington DC, 1989.

### MSM 413: GENERAL MEASURE AND INTEGRATION THEORY

**External Theory Examination: 70 Marks** 

**Internal Assessment: 30 Marks** 

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all taking four questions from each section. The candidate is required to attempt five questions selecting at least two questions from each section.

# **SECTION-I** (Four Ouestions)

Measures, some properties of measures, outer measures, extension of measures, uniqueness of extension, completion of a measure, the LUB of an increasingly directed family of measures. (Scope as in the Sections 3-6, 9-10 of Chapter 1 of the book 'Measure and Integration' by S.K.Berberian).

Measurable functions, combinations of measurable functions, limits of measurable functions, localization of measurability, simple functions (Scope as in Chapter 2 of the book 'Measure and Integration' by S.K.Berberian).

Measure spaces, almost everywhere convergence, fundamental almost everywhere, convergence in measure, fundamental in measure, almost uniform convergence, Egoroff's theorem, Riesz-Weyl theorem (Scope as in Chapter 3 of the book 'Measure and Integration' by S.K.Berberian).

Integration with respect to a measure: Integrable simple functions, non-negative integrable functions, integrable functions, indefinite integrals, the monotone convergence theorem, mean convergence (Scope as in Chapter 4 of the book 'Measure and Integration' by S.K.Berberian)

# **SECTION-II** (Four Questions)

Signed Measures: Absolute continuity, finite singed measure, contractions of a finite signed measure, purely positive and purely negative sets, comparison of finite measures, Lebesgue decomposition theorem, a preliminary Radon-Nikodym theorem, Hahn decomposition, Jordan decomposition, upper variation, lower variation, total variation, domination of finite signed measures, the Radyon-Nikodym theorem for a finite measure space, the Radon-Nikodym theorem for a  $\sigma$  - finite measure space (Scope as in Chapter 7 (except Section 53) of the book 'Measure and Integration' by S.K.Berberian).

Integration over locally compact spaces: continuous functions with compact support,  $G_{\delta}$ 's and  $F_{\sigma}$ 's, Baire sets, Baire function, Baire-sandwich theorem, Baire measure, Borel sets, Regularity of Baire measures, Regular Borel measures, Integration of continuous functions with compact support, Riesz-Markoff's theorem (Scope as in relevant parts of the sections 54-57, 60, 62, 66 and 69 of Chapter 8 of the book 'Measure and Integration' by S.K.Berberian)

Product Measures: Rectangles, Cartesian product of two measurable spaces, measurable rectangle, sections, the product of two finite measure spaces, the product of any two measure spaces, product of two σ - finite measure spaces; iterated integrals, Fubini's theorem, a partial converse to the Fubini's theorem (Scope as in Chapter 6 (except section 42) of the book 'Measure and Integration' by S.K.Berberian)

# **Recommended Text Book:**

S.K.Berberian: Measure and Integration, Chelsea Publishing Company, New York, 1965.

- H.L.Royden: Real Analysis, Prentice Hall of India, 3<sup>rd</sup> Edition, 1988. 1.
- G.de Barra: Measure Theory and Integration, Wiley Eastern Ltd., 1981. 2.

- 3.
- P.R.Halmos: Measure Theory, Van Nostrand, Princeton, 1950. I.K.Rana: An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 4. 1997.
- R.G.Bartle: The Elements of Integration, John Wiley and Sons, Inc. New York, 1966. 5.

#### **MSM 414: LINEAR PROGRAMMING**

**External Theory Examination: 70 Marks** 

**Internal Assessment: 30 Marks** 

**Time: 3 Hours** 

Note: Examiner is expected to set four questions from each section. Examinees are required to attempt five questions but at least two questions from each section.

## **Section-I** (four questions)

Simultaneous linear equations, Basic solutions, Linear transformations, Point sets, Lines and hyperplanes, Convex sets, Convex sets and hyperplanes, Convex cones, Restatement of the LP problem, Slack and surplus variables, Preliminary remarks on the theory of the simplex method, Reduction of any feasible solution to a basic feasible solution, Definitions and notations regarding LP problems. Improving a basic feasible solution, Unbounded solutions, Optimality conditions, Alternative optima, Extreme points and basic feasible solutions.

The simplex method, Selection of the vector to enter the basis, Degeneracy and breaking ties, Further development of the transformation formulas, The initial basic feasible solution-----artificial variables, Inconsistency and redundancy, Tableau format for simplex computations, Use of the tableau format, Conversion of a minimization problem to a maximization problem, Review of the simplex method.

The two-phase method for artificial variables, Phase I, Phase II, Numerical examples of the two-phase method, Requirements space, Solutions space, Determination of all optimal solutions, Unrestricted variables, Charnes' perturbation method regarding the resolution of the degeneracy problem.

# **Section-II (four questions)**

Selection of the vector to be removed, Definition of  $b(\clubsuit)$ . Order of vectors in  $b(\clubsuit)$ , Use of perturbation technique with simplex tableau format, Geometrical interpretation of the perturbation method. The generalized linear programming problem, The generalized simplex method, Examples pertaining to degeneracy, An example of cycling.

Revised simplex method: Standard Form I, Computational procedure for Standard Form I, Revised simplex method: Standard Form II, Computational procedure for Standard Form II, Initial identity matrix for Phase I, Comparison of the simplex and revised simplex methods, The product form of the inverse of a non-singular matrix. Alternative formulations of linear programming problems,

Dual linear programming problems, Fundamental properties of dual problems, Other formulations of dual problems, Complementary slackness, Unbounded solution in the primal, Dual simplex algorithm, Alternative derivation of the dual simplex algorithm, Initial solution for dual simplex algorithm, The dual simplex algorithm; an example, geometric interpretations of the dual linear programming problem and the dual simplex algorithm. A primal dual algorithm, Examples of the primal-dual algorithm. Transportation problem, its formulation and simple examples.

- 1. Linear Programming, (G.Hadley), Narosa Publishing House (1995)
- 2. Linear Programming: Methods and Applications, 4<sup>th</sup> Ed., (S.I. Gauss), McGraw-Hill, New York (1975)

### MSM 415: MATHEMATICAL ASPECTS OF SEISMOLOGY

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

**Time: 3 Hours** 

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **Section –I (Four Questions)**

Waves: General form of progressive waves, Harmonic waves, Plane waves, the wave equation. Principle of superposition. Progressive types solutions of wave equation. Stationary type solutions of wave equation in Cartesian, Cylindrical and Spherical coordinates systems. Equation of telegraphy. Exponential form of harmonic waves. D' Alembert's formula. Inhomogeneous wave equation. Dispersion: Group velocity, relation between phase velocity and group velocity.

(Relevant portions from the book "Waves" by Coulson & Jefferey)

Spherical waves. Expansion of a spherical wave into plane waves: Sommerfield's integral. Kirchoff's solution of the wave equation, Poissons's formula, Helmholtz's formula.

(Relevant portions from the book "Mathematical Aspects of Seismology" by Markus Bath).

Introduction to Seismology: Earthquakes, Location of earthquakes, Causes of Earthquakes, Observation of Earthquakes, Aftershocks and Foreshocks, Earthquake magnitude, Seismic moment, Energy released by earthquakes, Interior structure of the Earth.

# **Section-II (Four Questions)**

Reduction of equation of motion to wave equations. P and S waves and their characteristics. Polarization of plane P and S waves. Snell's law of reflection and refraction. Reflection of plane P and SV waves at a free surface. Partition of reflected energy. Reflection at critical angles.

Reflection and refraction of plane P, SV and SH waves at an interface. Special cases of Liquid-Liquid interface, Liquid-Solid interface and Solid-Solid interface.

Surface waves: Rayleigh waves, Love waves and Stoneley waves.

(Relevant portions from the book, "Elastic waves in Layered Media" by Ewing et al).

Two dimensional Lamb's problems in an isotropic elastic solid: Area sources and Line Sources in an unlimited elastic solid. A normal force acts on the surface of a semi-infinite elastic solid, tangential forces acting on the surface of a semi-infinite elastic solid.

Three dimensional Lamb's problems in an isotropic elastic solid: Area or Volume sources and Point sources in an unlimited elastic solid, Area or Volume source and Point source on the surface of semi-infinite elastic solid.

Haskell matrix method for Love waves in multilayered medium.

(Relevant portions from the book "Mathematical Aspects of Seismology" by Markus Bath)

#### **Recommended books:**

- 1. C.A. Coulson and A. Jefferey, Waves, Longman, New York, 1977.
- 2. M. Bath, *Mathematical Aspects of Seismology*, Elsevier Publishing Company, 1968.
- 3. W.M. Ewing, W.S. Jardetzky and F. Press, *Elastic Waves in Layered Media*, McGraw Hill Book Company, 1957.
- 4. C.M.R. Fowler, *The Solid Earth*, Cambridge University Press, 1990

#### **Reference books:**

1. P.M. Shearer, Introduction to Seismology, Cambridge University Press, (UK) 1999.

- 2. Jose Pujol, Elastic Wave Propagation and Generation in Seismology, Cambridge Universty Press, 2003.
- 3. Seth Stein and Michael Wysession, An Introduction to Seismology, Earthquakes and Earth Structure, Blackwell Publishing Ltd., 2003.
- 4. Aki, K. and P.G. Richards, Quantitative Seismology: theory and methods, W.H. Freeman, 1980.
- 5. Bullen, K.E. and B.A. Bolt, An Introduction to the Theory of Seismology, Cambridge Universty Press, 1985.

#### MSM 416: NON-COMMUTATIVE RINGS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all by taking four questions from each section. The examinee will be required to attempt five questions by selecting at least two questions from each section.

## **SECTION-I** (Four Questions)

Basic terminology and examples of non-commutative rings i.e. Hurwitz's ring of integral quaternions, Free k-rings. Rings with generators and relations. Hilbert's Twist, Differential polynomial rings, Group rings, Skew group rings, Triangular rings, D.C.C. and A.C.C. in triangular rings. Dedekind finite rings. Simple and semi-simple modules and rings. Spliting homomorphisms. Projective and Injective modules.

Ideals of matrix ring  $M_n(R)$ . Structure of semi simple rings. Wedderburn-Artin Theorem Schur's Lemma. Minimal ideals. Indecomposable ideals. Inner derivation δ. δ-simple rings. Amitsur Theorem on non-inner derivations. Jacobson radical of a ring R.Annihilator ideal of an R-module M. Jacobson semi-simple rings. Nil and Nilpotent ideals. Hopkins-Levitzki Theorem. Jacobson radical of the matrix ring  $M_n(R)$ . Amitsur Theorem on radicals. Nakayama's Lemma. Von Neumann regular rings. E. Snapper's Theorem. Amitsur Theorem on radicals of polynomial rings.

# **SECTION-II (Four Questions)**

Prime and semi-prime ideals. m-systems. Prime and semi-prime rings. Lower and upper nil radical of a ring R Amitsur theorem on nil radical of polynomial rings. Brauer's Lemma. Levitzki theorem on nil radicals. Primitive and semi-primitive rings. Left and right primitive ideals of a ring R. Density Theorem. Structure theorem for left primitive rings.

Sub-direct products of rings. Subdirectly reducible and irreducible rings. Birchoff's Theorem. Reduced rings. G.Shin's Theorem. Commutativity Theorems of Jacobson, Jacobson-Herstein and Herstein Kaplansky. Division rings. Wedderburn's Little Theorem. Herstein's Lemma. Jacobson and Frobenius Theorem. Cartan-Brauer-Hua Theorem. Herstein's Theorem.

(Section 1.1 to 1.26, Section 2.1 to 2.9, Section 3.1 to 3.19, Section 4.1 to 4.27, Section 5.1 to 5.10, Section 10.1 to 10.30, Section 11.1 to 11.20, Section 12.1 to 12.11 and Section 13.1 to 13.26 of the book "A First Course in Noncommutive Rings" by T.Y.Lam).

### **Recommended Books:**

- 1. T.Y.Lam A First Course in Noncommutive Rings, (Springer Verlag 1990)
- 2. I.N.Herstein Non-Commutative Rings carus monographs in Mathematics

Vol.15. Math Asso, of America 1968.

### MSM 417: WAVELET ANALYSIS

External Theory Examination: 70 Marks

Internal Assessment: 30 Marks

Time: 3 Hours

NOTE: The examiner is requested to set eight questions in all taking four questions from each section. The candidate is required to attempt five questions selecting at least two questions from each section.

# **SECTION-I** (Four Questions)

Fourier Transform: The finite Fourier transform, the circle group T, convolution to T,  $(L_1(T),+,*)$  as a Banach algebra, convolutions to products, convolution on T, the exponential form of Lebesgue's theorem, Fourier transform: trigonometric approach, exponential form, Basics/examples.

Fourier transform and residues, residue theorem for the upper and lower half planes, the Abel kernel, the Fourier map, convolution on R, inversion, exponential form, inversion, trigonometric form, criterion for convergence, continuous analogue of Dini's theorem, continuous analogue of Lipschitz's test, analogue of Jordan's theorem,

(C,1) summability for integrals, the Fejer-Lebesgue inversion theorem,the continuous Fejer Kernel, the

Fourier map is not onto, a dominated inversion theorem, criterion for integrability of  $\hat{f}$ 

Approximate identity for  $L_1(R)$ , Fourier Sine and Cosine transforms, Parseval's identities, the  $L_2$  theory, Parseval's identities for  $L_2$ , inversion theorem for  $L_2$  functions, the Plancherel theorem, A sampling theorem, the Mellin transform, variations.

(Scope of this section is as in relevant parts of Chapter 5 of the book "Fourier and Wavelet Analysis" by Bachman, Narici and Beckenstein)

### **SECTION-II** (Four Questions)

Discrete Fourier transform, the DFT in matrix form, inversion theorem for the DFT, DFT map as a linear bijection, Parseval's identities, cyclic convolution, Fast Fourier transform for  $N=2^k$ , Buneman's Algorithm, FFT for N=RC, FFT factor form. (Scope as in relevant parts of Chapter 6 of the book "Fourier and Wavelet Analysis" by Bachman, Narici and Beckenstein)

Wavelets: orthonormal basis from one function, Multiresolution Analysis, Mother wavelets yield Wavelet bases, Haar wavelets, from MRA to Mother wavelet, Mother wavelet theorem, construction of scaling function with compact support, Shannon wavelets, Riesz basis and MRAs, Franklin wavelets, frames, splines, the continuous wavelet transform. (Scope as in relevant parts of Chapter 7 of the book "Fourier and Wavelet Analysis" by Bachman, Narici and Beckenstein)

#### **Recommended Book**

G. Bachman, L. Narici and E. Beckenstein: Fourier and Wavelet Analysis, Springer, 2000

- 1. Hernandez and G. Weiss: A first course on wavelets, CRC Press, New York, 1996
- 2. C. K. Chui: An introduction to Wavelets, Academic Press, 1992
- 3. I. Daubechies: Ten lectures on wavelets, CBMS\_NFS Regional Conferences in Applied Mathematics, 61, SIAM, 1992
- 4. V. Meyer, Wavelets, algorithms and applications SIAM, 1993
- 5. M.V. Wickerhauser: Adapted wavelet analysis from theory to software, Wellesley, MA, A.K. Peters, 1994
- 6. D. F. Walnut: An Introduction to Wavelet Analysis, Birkhauser, 2002
- 7. K. Ahmad and F.A. Shah: Introduction to Wavelets with
- 8. Applications, World Education Publishers, 2013