SYLLABUS FOR PH.D. ENTRANCE TEST MATHEMATICS (2011-12)

ADVANCED ABSTRACT ALGEBRA -I

Zassenhaus's Lemma, Normal series. Isomorphic normal series, Scheiers Theorem, Composition Series. Jordan-Holder Theorem.
Commutators and their properties. Jacobi's Identity, Three subgroups lemma of P. Hall. Central series. Nilpotent groups. Upper and lower central series and their properties. Special properties of upper and lower central series of a nilpotent group. Invariant (normal) and chief series. Solvable groups. Derived series.
Prime fields,characteristic of a field. Field Extensions, Algebraic and transcendental extensions. Algebraically closed fields, splitting fields. Roots of unity Cyclotomic Polynomial $\phi_n(x)$. Finite fields. Normal extensions.
Separable and inseparable extensions. Perfect fields. Automorphisms of extensions. Galois extensions and Galois groups. Fundamental theorem of Galois theory. Radical extensions. Galois radical extensions. Solutions of polynomial equations by radicals. Insolvability of the general equation of degree $n \geq 5$ by radicals. Construction with ruler and compass.

Recommended Books :

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi, 1975.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nag Paul, Basic Abstract Algebra (2nd Edition), Cambridge University Press; Indian Edition, 1997.
3. Field Theory, Mathematical Pamphlet – 3 (T.I.F.R. Bombay)
4. I.D.Macdonald, Theory of Groups Oxford at the Clarendon Press(1968).

References :

1. M.Artin, Algebra, Prentice Hall of India, 1991.
2. I.S. Luther and I.B.S. Passi, Algebra, Vol. I- Groups, Vol. II-Rings, Narosa Publishing House (Vol.I-1996, Vol.II-1999).
3. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
4. I. Stewart, Galois Theory, 2^{nd} Edition, Chapman and Hall, 1989.
5. Surjeet Singh, Modern Algebra.

ADVANCED ABSTRACT ALGEBRA-II

Similarity of linear Transformations. Invariant subspaces. Reduction of a Linear Transformation (matrix) to a triangular form. Nilpotent transformations. Index of nilpotency. Invariants of a nilpotent linear transformation. A primary decomposition theorem. Jordan blocks and Jordan canonical forms. Cyclic modules relative to a Linear Transformation. Companion matrix of a monic polynomial. Rational Canonical forms. Elementary divisors of a Linear Transformation.

Idempotents. Finitely generated modules. Cyclic modules. The abelian groups $Hom_Z(M,N)$ and $Hom_R(M,N)$ and the rings $Hom_Z(M,M)$ and $Hom_R(M,M)$ Opposite ring. Simple modules. Schur's Lemma. Semi-simple modules (completely reducible modules). Free modules. Hom (A,B) where A and B are direct sums of finite number of submodules.

Noetherian and Artinian modules and rings. Hilbert basis theorem. Wedderburn-Artin theorem. Uniform modules, primary modules, and Noether-Lasker theorem. Smith normal form over a principal ideal domain and rank. Fundamental structure theorem for finitely generated modules over a Principal ideal domain and its application to finitely generated Abelian groups.

Recommended Texts :

1. I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi, 1975.
2. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, Basic Abstract Algebra (2nd Edition), Cambridge University Press; Indian Edition, 1997.

References :

1. M. Artin, Algebra, Prentice Hall of India, 1991.
2. Vivek Sahai and Vikas Bist, Algebra, Narosa Publishing House, 1999.
3. T.Y. Lam, Lectures on Modules and Rings, GTM, Vol. 189, Springer-Verlag, 1999.
4. Surjeet Singh, Modern Algebra.

REAL ANALYSIS-I

Definition and existence of Riemann Stieltjes integral, properties of the integral, integration and differentiation, the fundamental theorem of integral calculus, integration by parts, integration of vector-valued functions, Rectifiable curves (Scope as in Chapter 6 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition).

Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weirstrass M-test, Abel's test and Dirichlet's test for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann Stieltjes integration, uniform convergence and differentiation, existence of a real continuous nowhere differentiable function, equicontinous families of functions, Weierstrass approximation theorem(Scope as in Sections 7.1 to 7.27 of Chapter 7 of Principles of Mathematical Analysis by Walter Rudin, Third Edition).

Power Series : Uniqueness theorem for power series, Abel's and Tauber's theorem, Taylor's theorem (Scope as in Sections 8.1 to 8.5 of Chapter 8 of 'Principles of Mathematical Analysis' by Walter Rudin, Third Edition)

Functions of several variables : linear transformations, Derivative in an open subset of R^n, Chain rule, Partial derivatives, directional derivatives, the contraction principle, inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Derivatives of higher order, mean value theorem for real functions of two variables, interchange of the order of differentiation, Differentiation of integrals (scope as in relevant portions of Chapter 9 of 'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition).

Integration of differential forms : Partitions of unity, differential forms, stokes theorem (scope as in relevant portions of Chapter 10of 'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition).

Recommended Text :

'Principles of Mathematical Analysis' by Walter Rudin (3rd Edition) McGraw-Hill, 1976.

Reference Books :

1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.
2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975.
3. A.J. White, Real Analysis; an introduction. Addison-Wesley Publishing Co., Inc., 1968.
4. E. Hewitt and K. Stromberg. Real and Abstract Analysis, Berlin, Springer, 1969.
5.Serge Lang, Analysis I & II, Addison-Wesley Publishing Company Inc., 1969.

REAL ANALYSIS -II

Lebesgue outer measure, elementary properties of outer measure, Measurable sets and their properties, Lebesgue measure of sets of real numbers, algebra of measurable sets, Borel sets and their measurability, characterization of measurable sets in terms of open, closed, F and G sets, existence of a non-measurable set.

Lebesgue measurable functions and their properties, characteristic functions, simple functions, approximation of measurable functions by sequences of simple functions, measurable functions as nearly continuous functions, Borel measurability of a function, almost uniform convergence, Egoroff's theorem, Lusin's theorem, convergence in measure, F.Riesz theorem that every sequence which is convergent in measure has an almost everywhere convergent subsequence.

The Lebesgue Integral :

Shortcomings of Riemann integral, Lebesgue integral of a bounded function over a set of finite measure and its properties, Lebsegue integral as a generalization of the Riemann integral, Bounded convergence theorem, Lebesgue theorem regarding points of discontinuities of Riemann integrable functions, integral of a non negative function, Fatou's lemma, Monotone convergence theorem, integration of series, the general Lebesgue integral, Lebesgue convergence theorem.

Differentiation and Integration :

Differentiation of monotone functions, Vitali's covering lemma, the four Dini derivatives, Lebesgue differentiation theorem, functions of bounded variation and their representation as difference of monotone functions, Differentiation of an integral, absolutely continuous functions, convex functions, Jensen's inequality.

The L^p spaces

The L^p spaces, Minkowski and Holder inequalities, completeness of L^p spaces, Bounded linear functionals on the L^p spaces, Riesz representation theorem.

Recommeded Text :

'Real Analysis' by H.L.Royden (3rd Edition) Prentice Hall of India, 1999.

Reference Books :

1. G.de Barra, Measure theory and integration, Willey Eastern Ltd.,1981.
2. P.R.Halmos, Measure Theory, Van Nostrans, Princeton, 1950.
3. I.P.Natanson, Theory of functions of a real variable, Vol. I, Frederick Ungar Publishing Co., 1961.
4. R.G.Bartle, The elements of integration, John Wiley & Sons, Inc.New York, 1966.
5. K.R.Parthsarthy, Introduction to Probability and measure, Macmillan Company of India Ltd.,Delhi, 1977.

P.K.Jain and V.P.Gupta, Lebesgue measure and integration, New age International (P) Ltd., Publishers, New Delhi, 1986.

TOPOLOGY-I

Countable set and uncountable set, Cardinal number/cardinality of a set, Schroeder-Berstein theorem, Cantor's theorem; the concepts of p.o.sets, infimum,supremum and maximal element in a p.o.set, Axiom of choice, Zorn's Lemma, Well-Ordering theorem

and Continuum hypothesis (Statements only). (Scope as in theorems 15-20, Chapter O of Kelley's book given at Sr. No. 1).
Definition and examples of topological spaces, Neighbourhoods, Neighbourhood system of a point and its properties, Interior point and interior of a set, interior as an operator and its properties, definition of a Closed set as complement of an open set, limit point (accumulation point) of a set, derived set of a set, definition of closure of a set as union of the set and its derived set, Adherent point (Closure point) of a set , closure of a set as set of adherent (closure) points, properties of closure, closure as an operator and its properties, boundary of a set, Dense set, a characterization of dense set.
Base for a topology and its characterization, Base for Neighbourhood system, Sub-base for a topology.
Relative (induced) Topology and subspace of a topological space. Alternate methods of defining a topology using 'properties' of 'Neighbourhood system', 'Interior Operator', 'Closed sets', Kuratowski closure operator and 'base'.
First countable, Second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and a second countable space, Lindelof theorems.
Comparison of Topologies on a set, about intersection and union of topologies, infimum and supremum of a collection of topologies on a set, the collection of all topologies on a set as a complete lattice.
Definition, examples and characterisations of continuous functions, composition of continuous functions, Open and closed functions, Homeomorphism, embedding.
Tychonoff product topology in terms of standard (defining) subbase, projection maps, their continuity and openness, Characterisation of product topology as the smallest topology with projections continuous, continuity of a function from a space into a product of spaces, countability and product spaces.
T_0 ,T_1,T_2,Regular and T_3 separation axioms, their characterization and basic properties i.e. hereditary property of T_0 ,T_1,T_2, Regular and T_3 spaces, and productive property of T_1 and T_2 spaces.
Quotient topology w.r.t. a map, Continuity of function with domain a space having quotient topology, About Hausdorffness of quotient space (scope as in theorems 8-11, Chapter 3 of Kelley's book given at Sr.No.1)
Completely regular and Tychonoff ($T_{3\,1/2}$), spaces, their hereditary and productive properties. Embedding lemma, Embedding theorem (Scope as in theorems 5-7, Chapter 4 of Kelley's book given at Sr. No. 1).
Normal and T_4 spaces : Definition and simple examples, Urysohn's Lemma, complete regularity of a regular normal space, T_4 implies Tychonoff, Tietze's extension theorem.

Separated subsets of a topological space, definition of separation of a topological space in terms of separated sets, characterizations of separation of a topological space, definition of connectedness in terms of separation, Characterisations of connectedness,definition of a connected subset of a topological space as a connected subspace, the definition of separation of a subset in the subspace and in the main space, their relation, properties of connected subsets, Continuity and connectedness,

Connectedness and product spaces, components and locally connected spaces. (Scope as in theorems 23.1-23.6, 25.1 and 25.3, chapter 3 of the book given at Sr.No. 2).

Books :

1. Kelley, J.L. : General Topology.

2. Munkres, J.R. : Topology, Second Edition, Prentice Hall of India/ Pearson.

TOPOLOGY -II

Definition and examples of filters on a set, Collection of all filters on a set as a p.o. set, finer filter, methods of generating filters/finer filters,Ultra filter (u.f.) and its characterisations, Ultra Filter Principle (UFP) i.e. Every filter is contained in an ultra filter. Image of filter under a function.

Convergence of filters : Limit point (Cluster point) and limit of a filter and relationship between them, Continuity in terms of convergence of filters, Hausdorffness and filter convergence.

Convergence of sequences in topological spaces and in first axiom topological spaces, Nets in topological spaces. Convergence of nets, Hausdorffness and convergence of nets, Subnets and cluster points, canonical way of converting nets to filters and vice versa, their convergence relations. (Scope as in theorem 2-3, 5-8 chapter 2 of Kelley's book given at Sr.No. 1).

Compactness : Definition and examples of compact spaces, definition of a compact subset as a compact subspace, relation of open cover of a subset of a topological space in the sub-space with that in the main space, compactness in terms of finite intersection property (f.i.p.), compactness and net convergence, continuity and compact sets, compactness and separation properties, Closedness of compact subset, closeness of continuous map from a compact space into a Hausdorff space and its consequence, Regularity and normality of a compact Hausdorff space.

Compactness and filter convergence, Convergence of filters in a product space, compactness and product space. Tychonoff product theorem using filters, Tychonoff space as a subspace of a compact Hausdorff space and its converse, local compactness, compactification and Hausdorff compactification one point compactification, Stone-Cech compactification, (Scope as in theorems 1,2,7-12,15,17,18, 21-24, chapter 5 of Kelley's book given at Sr. No. 1).

Definition and examples of metrisable spaces, examples of non-metrisable spaces, metrisability of product of countably many metrisable spaces,metrisability of a sub-space of a metrisable space, normality of a regular space which is second axim or Lindelof, Urysohn's metrisation theorem.

Locally finite family, its equivalent forms, countabily locally finite family, refinement, open refinement, closed refinement of a family, existence of countably locally finite open covering refining a given open covering of a metrizable space, Nagata-Smirnov metrization theorem, Paracompactness, normality of paracompact Hausdorff space, paracompactness of a metrizable space and of regular Lindelof space, Smirnov metrization theorem.(Scope as in theorems 39.1-39.2, 40.3, 41.1-41.5 and 42.1, chapter 6 of the book given at Sr.No. 2).

Homotopy relation of paths in topological spaces and of paths having common initial points and common final points, these two relations as equivalence relations, product of paths, operation of product of homotopy classes of paths and its properties, Fundamental group, change of base point, induced homomorphism of continuous map. Covering maps and covering spaces, Fundamental group of circle (Scope as in theorems, 51.1-51.2, 52.1-52.4,53.1-53.3 and 54.1-54.5, chapter 9 of the book given at Sr.No. 2).

Books :

1. Kelley, J.L. : General Topology.
2. Munkres, J.R. : Topology, Second Edition, Prentice Hall of India/ Pearson.

COMPLEX ANALYSIS -I

Power series, its convergence, radius of convergence, examples, sum and product, differentiability of sum function of power series, property of a differentiable function with derivative zero. expz and its properties, logz, power of a complex number (z), their branches with analyticity.
Path in a region, smooth path, p.w. smooth path, contour, simply connected region, multiply connected region, bounded variation, total variation, complex integration, Cauchy-Goursat theorem, Cauchy theorem for simply and multiply connected domains. Index or winding number of a closed curve with simple properties. Cauchy integral formula. Extension of Cauchy integral formula for multiple connected domain. Higher order derivative of Cauchy integral formula. Gauss mean value theorem Morera's theorem. Cauchy's inequality.
Zeros of an analytic function, entire function, radius of convergence of an entire function, Liouville's theorem, Fundamental theorem of algebra, Taylor's theorem, Maximum modulus principle, Minimum modulus principle. Schwarz Lemma.
Singularity, their classification, pole of a function and its order.
Laurent series, Cassorati – Weiertrass theorem , Meromorphic functions, The argument principle, Rouche's theorem, inverse function theorem, Residue : Cauchy residue theorem and its use to calculate certain integrals, definite integral, integral of the type $^{\infty}\int f(x)$ sinmx dx or $\int^{\infty} f(x)$ cosmx dx, poles on the real axis, many valued functions.
Bilinear transformation, their properties and classification. Definition and examples of conformal mapping.

Books recommended :

1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
3. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
3. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4. Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
5. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
6. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

COMPLEX ANALYSIS -II

Spaces of analytic functions and their completeness, Hurwitz's theorem, Montel's theorem, Riemann mapping theorem, infinite products, Weierstrass factorization theorem, Factorization of sine function, Gamma function and its properties, functional equation for gamma function, Integral version of gamma function, Reimann-zeta function, Riemann's functional equation, Runge's theorem, Mittag-Leffler's theorem..

Analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation along a curve, Power series method of analytic continuation , Schwarz reflection principle, Monodromy theorem and its consequences. Harmonic function as a disk, Poisson's Kernel, Dirichlet problem for a unit disk.

Harnack's inequality, Harnack's theorem, Dirichlet problem for a region, Green's function, Canonical product, Jensen's formula, Poisson-Jensen formula, Hadamard's three circle theorem, Order of an entire function, Exponent of convergence, Borel theorem, Hadamard's factorization theorem. The range of an analytic function, Bloch's theorem, Little-Picard theorem, Schottky's theorem, Montel-Carathedory theorem, Great Picard theorem.

Univalent functions, Bieberbach's conjecture (Statement only) and 1/ 4 theorem.

Books recommended :

1. Ahlfors, L.V., Complex Analysis. McGraw-Hill Book Company, 1979.
2. Churchill, R.V. and Brown, J.W., Complex Variables and Applications McGraw Hill Publishing Company, 1990.
3. Conway, J.B., Functions of One complex variables Narosa Publishing, 2000.

Reference Books :

1. Priestly, H.A., Introduction to Complex Analysis Claredon Press, Orford, 1990.
2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.
3. D.Sarason, Complex Function Theory, Hindustan Book Agency, Delhi, 1994.
4. Mark J.Ablewitz and A.S.Fokas, Complex Variables : Introduction & Applications, Cambridge University Press, South Asian Edition, 1998.
5. E.C.Titchmarsn, The Theory of Functions, Oxford University Press, London.
6. S.Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 1997.

<u>Differential Equation -I</u>

Preliminaries: Initial value problem and equivalent integral equation, ε-approximate solution, equicontinuous set of functions.
Basic theorems: Ascoli- Arzela theorem, Cauchy –Peano existence theorem and its corollary. Lipschitz condition. Differential inequalities and uniqueness- Gronwall's inequality. Successive approximations. Picard-Lindelöf theorem. Continuation of solution, Maximal interval of existence, Extension theorem. Kneser's theorem (statement only)
System of differential equations, the n-th order equation. Dependence of solutions on initial conditions and parameters: Preliminaries, continuity and differentiability.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)
Linear differential systems: Definitions and notations. Linear homogeneous systems; Fundamental matrix, Adjoint systems, reduction to smaller homogeneous systems. Non-homogeneous linear systems; variation of constants. Linear systems with constant coefficients. Linear systems with periodic coefficients; Floquet theory.
(Relevant portions from the book of 'Theory of Ordinary Differential Equations' by Coddington and Levinson)
Higher order equations: Linear differential equation (LDE) of order n; Linear combinations, Linear dependence and linear independence of solutions. Wronskian theory: Definition, necessary and sufficient condition for linear dependence and linear independence of solutions of homogeneous LDE. Abel's identity, Fundamental set, More Wronskian theory. Reduction of order. Non-homogeneous LDE. Variation of parameters. Adjoint equations, Lagrange's Identity, Green's formula. Linear equation of order n with constant coefficients. (Relevant portions from the books of 'Theory of Ordinary Differential Equations' by Coddington and Levinson and the book 'Differential Equations' by S.L. Ross)
Maximal and Minimal solutions. Differential inequalities. A theorem of Wintner. Uniqueness theorems: Kamke's theorem, Nagumo's theorem and Osgood theorem. (Relevant portions from the book 'Ordinary Differential Equations' by P. Hartman)

Referneces:

1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill , 2000.
2. S.L. Ross, *Differential Equations*, John Wiley & Sons,
3. P. Hartman, *Ordinary Differential Equations*, John Wiley & Sons NY, 1971.
4. G. Birkhoff and G.C. Rota, *Ordinary Differential Equations*, John Wiley & Sons, 1978.
5. G.F. Simmons, *Differential Equations*, Tata McGraw-Hill , 1993.
6. I.G. Petrovski, *Ordinary Differential Equations*, Prentice-Hall, 1966.
7. D. Somasundaram, *Ordinary Differential Equations, A first Course*, Narosa Pub., 2001.

S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations*, Tata McGraw-Hill , 2006.

<u>DIFFERENTIAL EQUATIONS -II</u>

_Linear second order equations: Preliminaries, self adjoint equation of second order, Basic facts, superposition principle, Riccati's equation, Prüffer transformation, zero of a solution, oscillatory and non-oscillatory equations. Abel's formula. Common zeros of solutions and their linear dependence.
Sturm theory: Sturm separation theorem, Sturm fundamental comparison theorem and its corollaries. Elementary linear oscillations.
(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Textbook of Ordinary Differential Equations' by Deo et al.)
_Autonomous systems: the phase plane, paths and critical points, Types of critical points; Node Center, Saddle point, Spiral point. Stability of critical points. Critical points and paths of linear systems: basic theorems and their applications. Critical points and paths of non-linear systems: basic theorems and their applications. Liapunov function. Liapunov's direct method for stability of critical points of non-linear systems.
Limit cycles and periodic solutions: Limit cycle, existence and non-existence of limit cycles, Benedixson's non-existence criterion. Half-path or Semiorbit, Limit set, Poincare-Benedixson theorem. Index of a critical point.
(Relevant portions from the book 'Differential Equations' by S.L. Ross and the book 'Theory of Ordinary Differential Equations' by Coddington and Levinson)
_Second order boundary value problems(BVP): Linear problems; periodic boundary conditions, regular linear BVP, singular linear BVP; non-linear BVP. Sturm-Liouville BVP: definitions, eigen value and eigen function. Orthogonality of functions, orthogonality of eigen functions corresponding to distinct eigen values. Green's function. Applications of boundary value problems. Use of Implicit function theorem and Fixed point theorems for periodic solutions of linear and non-linear equations.
(Relevant portions from the book 'Textbook of Ordinary Differential Equations' by Deo et al.)

Referneces:

1. E.A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Tata McGraw-Hill , 2000.
2. S.L. Ross, *Differential Equations*, John Wiley & Sons,
3. S.G. Deo, V. Lakshmikantham and V. Raghavendra, *Textbook of Ordinary Differential Equations*, Tata McGraw-Hill , 2006.
4. P. Hartman, *Ordinary Differential Equations*, John Wiley & Sons NY, 1971.
5. G. Birkhoff and G.C. Rota, *Ordinary Differential Equations*, John Wiley & Sons, 1978.
6. G.F. Simmons, *Differential Equations*, Tata McGraw-Hill , 1993.
7. I.G. Petrovski, *Ordinary Differential Equations*, Prentice-Hall, 1966.
8. D. Somasundaram, *Ordinary Differential Equations, A first Course*, Narosa Pub., 2001.

Functional Analysis

Normed linear spaces, Banach spaces and examples, subspace of a Banach space, completion of a normed space, quotient space of a normed linear space and its completeness, product of normed spaces, finite dimensional normed spaces and subspaces, equivalent norms, compactness and finite dimension, F.Riesz's lemma. Bounded and continuous linear operators, differentiation operator, integral operator, bounded linear extension, linear functionals, bounded linear functionals, continuity and boundedness, definite integral, canonical mapping, linear operators and functionals on finite dimensional spaces, normed spaces of operators, dual spaces with examples.
(Scope of this section is as in relevant parts of Chapter 2 of 'Introductory Functional Analysis with Applications' by E.Kreyszig)
_Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces, application to bounded linear functionals on C[a,b], Riesz-representation theorem for bounded linear functionals on C[a,b], adjoint operator, norm of the adjoint operator. Reflexive spaces, uniform boundedness theorem and some of its applications to the space of polynomials and fourier series, strong and weak convergence, weak convergence in *l*, convergence of sequences of operators, uniform operator convergence, strong operator convergence, weal operator convergence, strong and weak* convergence of a sequence of functionals.
Open mapping theorem, bounded inverse theorem, closed linear operators, closed graph theorem, differential operator, relation between closedness and boundedness of a linear operator.
(Scope of this section is as in relevant parts of Chapter 4(except 4.10 and 4.11) of 'Introductory Functional Analysis with Applications' by E.Kreyszig)
_Inner product spaces, Hilbert spaces and their examples, pythagorean theorem, Apolloniu's identity, Schwarz inequality, continuity of innerproduct, completion of an inner product space, subspace of a Hilbert space, orthogonal complements and direct sums, projection theorem, characterization of sets in Hilbert spaces whose space is dense. Orthonormal sets and sequences, Bessel's inequality, series related to orthonormal sequences and sets, total(complete) orthonormal sets and sequences, Parseval's identity, separable Hilbert spaces.Representation of functionals on Hilbert spaces, Riesz representation theorem for bounded linear functionals on a Hilbert space, sesquilinear form, Riesz representation theorem for bounded sesquilinear forms on a Hilbert space. Hilbert adjoint operator, its existence and uniqueness, properties of Hilbert adjoint operators, self adjoint, unitary, normal, positive and projection operators.
(Scope of this section is as in relevant parts of Chapter 3 (except 3.7) and sections 9.3 to 9.6 of Chapter 9 of 'Introductory Functional Analysis with Applications' by E.Kreyszig.

Recommended Text:

E.Kreyszig: Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1978.

References:

1. G.F.Simmons: Introduction to Topology and Modern Analysis, McGraw Hill Book Co.,New York, 1963.

2. C.Goffman and G.Pedrick: First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.
3. G.Bachman and L.Narici, Functional Analysis, Academic Press, 1966.
4. L.A.Lustenik and V.J.Sobolev, Elements of Functional Analysis, Hindustan Publishing Corporation, New Delhi, 1971.
5. J.B.Conway: A Course in Functional Analysis, Springer-Verlag, 1990.
6. P.K.Jain, O.P.Ahuja and Khalil Ahmad: Functional Analysis, New Age International(P) Ltd. & Wiley Eastern Ltd., New Delhi, 1997.

Analytical Mechanics and Calculus of Variations

Motivating problems of calculus of variations, shortest distance. Minimum surface of revolution. Brachistochrone problem, Isoperimetric problem. Geodesic. Fundamental Lemma of calculus of variation. Euler's equation for one dependent function of one and several independent variables, and its generalization to (i) Functional depending on 'n' dependent functions, (ii) Functional depending on higher order derivatives. Variational derivative, invariance of Euler's equations, natural boundary conditions and transition conditions, Conditional extremum under geometric constraints and under integral constraints . Variable end points.

Free and constrained systems, constraints and their classification. Generalized coordinates. Holonomic and Non-Holonomic systems. Scleronomic and Rheonomic systems. Generalized Potential, Possible and virtual displacements,ideal constraints. . Lagrange's equations of first kind, Principle of virtualdisplacements D'Alembert's principle, HolonomicSystems independent coordinates, generalized forces, Lagrange's equations of second kind. Uniqueness of solution. Theorem on variation of total Energy. Potential, Gyroscopic and dissipative forces, Lagrange's equations for potential forces equation for conservative fields.

Hamilton's variables. Don kin's theorem. Hamilton canonical equations. . Routh's equations. Cyclic coordinates Poisson's Bracket. Poisson's Identity. Jacobi-Poisson theorem. Hamilton's Principle, second form of Hamilton's principle. Poincare-Carton integral invariant. Whittaker's equations. Jacobi's equations. Principle of least action Statement of Lee Hwa Chung's theorem.

Canonical transformations and the Hamilton-Jacobi equation. Jacobi theorem. Method of separation of variables. Testing the Canonical character of a transformation. Lagrange brackets. Condition of canonical character of as transformation in terms of Lagrange brackets and Poisson brackets. Invariance of Lagrange brackets and Poisson brackets under canonical transformations.

Books:

1. F. Gantmacher, Lectures in Analytic Mechanics, Khosla Publishing House, New Delhi.
2. H. Goldstein, Classical Mechanics (2nd edition), Narosa Publishing House, New Delhi.
3. I.M. Gelfand and S.V. Fomin, Calculus of Variations, Prentice Hall.
4. Francis B. Hilderbrand, Methods of applied mathematics , Prentice Hall,
5. Narayan Chandra Rana & Pramod Sharad Chandra Joag. Classical Mechanics, Tata McGraw Hill, 1991.
6. Louis N. Hand and Janet D. Finch, Analytical Mechanics, Cambridge University Press, 1998.

Computer Programming (Theory)

Numerical constants and variables; arithmatic expressions; input/output; conditional flow; looping.

Logical expressions and control flow; functions; subroutines; arrays; format specifications; strings; array arguments.

Derived data types; processing files; pointers; modules; FORTRAN 90 features; FORTRAN 95 features.

Recommended Text

V. Rajaraman : Computer Programming in FORTRAN 90 and 95; Prentice-Hall of India Pvt. Ltd., New Delhi, 1997.

References

1. V. Rajaraman : Computer Programming in FORTRAN 77, Prentice-Hall of India Pvt. Ltd., New Delhi, 1984.
2. J. F. Kerrigan : Migrating to FORTRAN 90, Orielly Associates, CA, USA, 1993.
3. M. Metcalf and J. Reid : FORTRAN 90/95 Explained, OUP, Oxford, UK, 1996.

General Measure and Integration Theory

Measures, some properties of measures, outer measures, extension of measures, uniqueness of extension, completion of a measure, the LUB of an increasingly directed family of measures.(Scope as in the Sections 3-6, 9-10 of Chapter 1 of the book 'Measure and Integration' by S.K.Berberian).

Measurable functions, combinations of measurable functions, limits of measurable functions, localization of measurability, simple functions (Scope as in Chapter 2 of the book 'Measure and Integration' by S.K.Berberian).

Measure spaces, almost everywhere convergence, fundamental almost everywhere, convergence in measure, fundamental in measure, almost uniform convergence, Egoroff's theorem, Riesz-Weyl theorem (Scope as in Chapter 3 of the book 'Measure and Integration' by S.K.Berberian).

_Integration with respect to a measure: Integrable simple functions, non-negative integrable functions, integrable functions, indefinite integrals, the monotone convergence theorem, mean convergence (Scope as in Chapter 4 of the book 'Measure and Integration' by S.K.Berberian)

Product Measures: Rectangles, Cartesian product of two measurable spaces, measurable rectangle, sections, the product of two finite measure spaces, the product of any two measure spaces, product of two σ - finite measure spaces; iterated integrals, Fubini's theorem, a partial converse to the Fubini's theorem (Scope as in Chapter 6 (except section 42) of the book 'Measure and Integration' by S.K.Berberian)

_Signed Measures: Absolute continuity, finite singed measure, contractions of a finite signed measure, purely positive and purely negative sets, comparison of finite measures, Lebesgue decomposition theorem, a preliminary Radon-Nikodym theorem, Hahn decomposition, Jordan decomposition, upper variation, lower variation, total variation, domination of finite signed measures, the Radyon-Nikodym theorem for a finite measure space, the Radon-Nikodym theorem for a σ - finite measure space (Scope as in Chapter 7 (except Section 53) of the book 'Measure and Integration' by S.K.Berberian).
Integration over locally compact spaces: continuous functions with compact support, $G\delta$'s and $F\sigma$'s, Baire sets, Baire function, Baire-sandwich theorem, Baire measure, Borel sets, Regularity of Baire measures, Regular Borel measures, Integration of continuous functions with compact support, Riesz-Markoff's theorem (Scope as in relevant parts of the sections 54-57,60,62,66 and 69 of Chapter 8 of the book 'Measure and Integration' by S.K.Berberian)

<u>Recommended Text:</u>

S.K.Berberian: Measure and Integration, Chelsea Publishing Company, New York, 1965.

References:

1. H.L.Royden: Real Analysis, Prentice Hall of India, 3rd Edition, 1988.
2. G.de Barra: Measure Theory and Integration, Wiley Eastern Ltd.,1981.
3. P.R.Halmos: Measure Theory, Van Nostrand, Princeton, 1950.
4. I.K.Rana: An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997.
5. R.G.Bartle: The Elements of Integration, John Wiley and Sons, Inc. New York, 1966.

Partial Differential Equations

PDE of k^{th} order: Definition, examples and classifications. Transport equations homogeneous and non-homogeneous, Initial value problem. Laplace's Equations: Fundamental solutions, harmonic functions and their properties, Mean value Formulas, Poisson,s equation and its solution,Strong maximum principle, uniqueness, local estimates for harmonic functions, Liouville,s theorem, Harnack's inequality. Green's function and its derivation, representation formula using Green's function, symmetry of Green's function, Green's function for a half space and for a ball. Energy methods: uniqueness, Drichlet's principle.

Heat Equations: Physical interpretation, fundamental solution. Integral of fundamental solution , solution of initial value problem, non-homogeneous problem, Mean value formula for heat equation, strong maximum principle and uniqueness. Properties of solutions, Energy methods.
Wave equation- Physical interpretation , Solution by spherical means, solution for n=1, d'Alemberts formula and its applications , reflection method, Euler-Poisson_Darboux equation, Kirchhoff's and Poisson's formulas, Solution of non –homogeneous wave equation. Energy method.

Non-linear first order PDE- complete integrals, envelopes Characteristics Hamilton Jacobi equations (calculus of variations Hamilton's ODE, Legendre Transform, Hopf-Lax formula, weak solutions, Uniqueness) Conservative Laws (Shocks, entropy condition, Lax-Oleinik formula., weak solutions uniqueness. Riemann's problem, long time behaviuor.

Representation of Solutions- Separation of variables, Similarity solutions (Plane and traveling waves, solitones, similarity under Scaling) Fourier Transform, Laplace Transform, Hop-cole Transform, Potential functions. Hodograph and Legendre transforms.

Books:

1L.C. Evans, Partial Differential Equations, Graduate Studies in

2 Books with the above title by I.N. Snedden, F. John, P. Prasad and R. Ravindran, Amarnath etc.

Computer Programming (Practical)

Computer programs based on

1. Solutions of simultaneous linear equations.
2. Solution of algebraic / transcendental equations.
3. Inversion of matrices
4. Numerical differentiation and integration
5. Solution of ordinary differential equations
6. Statistical problems on measure of central tendency, dispersion, correlation and regression.
7. Fitting of curves by least square method.

Objective type Questions

1. Let G be a non-Abelian nilpotent group of order 540. Then order of its centre may be equal to :

   (1) 135 (2) 20

   (3) 108 (4) 30

2. Let G be a non-Abelian solvable group of order 200 and A is a minimal normal subgroup of G. Then order of A may be equal to :

   (1) 10 (2) 20

   (3) 25 (4) 40

3. Which of the following is not on F_σ-set :

   (1) A closed set (2) A countable set

   (3) An open set (4) None of these

4. Choose the incorrect statement :

   (1) A countable union of measurable set is a measurable set

   (2) Every Borel set is not measurable

   (3) Every set of positive measure contains a non-measurable set

   (4) None of the above

5. Let τ be a topology on **R** such that (**R**, τ) is T_1. Then τ is :

   (1) Cofinite topology (2) Usual topology

   (3) Discrete topology (4) None of these

6. Let X = **R-Q** and τ usual induced topology on X. Then (X, τ) :

(1) is Lindelof (2) is compact and Hausdorff

(3) is compact and T_1 (4) Has no countable dense set

7. The zeros of zeta function ζ(z) on the real line z = 1 are :

(1) 1 (2) infinte

(3) none (4) 2

8. Bessel's equation is :

(1) non-oscillatory

(2) oscillatory

(3) can be both oscillatory and non-oscillatory depending upon the order

(4) none of the above

9. Which is the correct statement ?

(1) The product of two projections is again a projection

(2) The sum of two projections is again a projection

(3) The difference of two projections is again a projection

(4) None of the above

10. A measure on a ring is :

(1) not monotone

(2) not continuous from below

(3) conditionally continuous from above

(4) countably additive.

Subjective Type Questions

11. If the zeta function can be defined to be mermorphic in the plane with only a simple pole at z = 1 and Res(ζ; 1) = 1, then show that for z ≠ 1, ζ satisfies Riemann's functional

equation.

12. If x(t) is a solution of I.V.P., x′(t) = f(t, x), x(τ) = ξ on (a, b), then prove that x(t) can be extended as a solution on the maximal interval of existence. What will happen if t→ either end point of the maximal interval of existence ?

13. a) Let A and B be two non-empty subsets of an infinite T_3 space. Find out full justification whether there exists disjoint open sets containing A and B.

b) Show that on every infinite set there exists a filter which is contained in at least two ultrafilters.

c) Find the collection of all real valued continuous fuctions defined on R with cofinite topology. The topology of the codomain is the usual topology.

14. State what Brachitochrome problem is? Formulate it, mathematically and hence solve it.

Explain the reason why the solution is not a straight line