Question Booklet Sr. No.

Roll No. ....

OMR Sheet No. ....

Ph.D. MATHEMATICS ENTRANCE TEST, FEB 2024

Time: 2 Hours

Maximum Marks: **100**

Number of Pages in this Booklet: 20	Number of Questions in this Booklet: **100**

INSTRUCTIONS FOR THE CANDIDATES

i. Check this booklet carefully for the sequence of pages and questions. If it is defective due to pages/questions missing or not in serial order or any other discrepancy it should be got replaced immediately from the invigilator within the period of 5 minutes. Afterwards neither the Question Booklet will be replaced nor any extra time will be given.

ii. After this verification write your Roll number and OMR Sheet Number on this Question Booklet.

iii. **Use only Black or Blue ball point pen**.

iv. This paper consists of **100** multiple choice type questions. Each question has 4 alternative answers (a), (b), (c) and (d). **Only one of these alternative answers is correct**. You are required to darken completely the circle of correct answer in the OMR Sheet.

v. There will be no negative marking.

vi. Do not write anything other than relevant entries or put any mark on any part of the OMR Sheet, which may disclose your identity, otherwise you will render yourself liable to disqualification.

vii. Use of electronic gadgets such as pager, cell phone, calculator and log table etc. is prohibited.

viii. Rough Work may be done in the end of this booklet.

ix. You have to **return the OMR Sheet** to the invigilator at the end of the examination compulsory.

1. Let $c \in \mathbb{Z}_3$ be such that $\frac{\mathbb{Z}_3[x]}{<x^3+cx+1>}$ is a field. Then c is equal to
   (a) 1
   (b) 2
   (c) 0
   (d) does not exist

2. Let $p(x) = 9x^5 + 10x^3 + 5x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in $\mathbb{Q}[x]$. Then, over $\mathbb{Q}$
   (a) Both $p(x)$ and $q(x)$ are irreducible
   (b) $p(x)$ is reducible but $q(x)$ is irreducible
   (c) $p(x)$ is irreducible but $q(x)$ is reducible
   (d) Both $p(x)$ and $q(x)$ are reducible

3. Let $\mathbb{F}$ be a field of order 32. Then the number of non zero solutions of the form $(a, b) \in \mathbb{F}\ X\ \mathbb{F}$ of the equation $x^2 + xy + y^2 = 0$, is equal to
   (a) one solution
   (b) two solutions
   (c) three solutions
   (d) no solution

4. Let G be a nonabelian group of order 125. Then the total number of elements in $Z(G)$ equals
   (a) 1
   (b) 5
   (c) 25
   (d) 125

5. In the permutation group S_6, the number of elements of order 8 is
   (a) 0
   (b) 1
   (c) 2
   (d) 3

6. Let $\mathbb{F}$ be a field with 7^6 elements and let $\mathbb{K}$ be a subfield of $\mathbb{F}$ with 49 elements. Then the dimension of $\mathbb{F}$ as a vector space over $\mathbb{K}$ is
   (a) 1
   (b) 2
   (c) 3
   (d) 6

7. Let ω be a primitive cube root of unity. Then the degree of the field extension $\mathbb{Q}(i, \sqrt{3}, \omega)$ over $\mathbb{Q}$ is
   (a) 1
   (b) 2
   (c) 4
   (d) 8

8. Let G be a noncyclic group of order 57. Then the number of elements of order 3 in G is

(a) 1
(b) 3
(c) 19
(d) 38

9. Consider the following statements:

P: An abelian group G has a composition series if and only if G is finite.
R: If a cyclic group has exactly one composition series, then it is a $p-$group.

(a) Both P and R are true
(b) P is true and R is false
(c) P is false and R is true
(d) Both P and R are false

10. Let G be a group of order 15 and H be a group of order 35.

(a) Both G and H are cyclic groups
(b) G is cyclic and H is not cyclic
(c) G is not cyclic and H is cyclic
(d) Both G and H are not cyclic groups

11. Let A, B and C be $R-$submodules of an $R-$module M, then

$$A \cap (B + C) = (A \cap B) + (A \cap C)$$

(a) always true
(b) never true
(c) true if $B \subset A$
(d) cannot be determined

12. The degree of extension of the splitting field of $x^3 - 2 \in \mathbb{Q}[x]$ over $\mathbb{Q}$ is

(a) 1
(b) 2
(c) 3
(d) 6

13. A regular $n-$gon(polygon with n sides) is constructible if n is

(a) 7
(b) 9
(c) 11
(d) 17

14. $\mathbb{Z}$ (set of integers)as a $\mathbb{Z}$ module is

(a) Both Artinian and Noetherian
(b) Artinian but not Noetherian
(c) Noetherian but not Artinian
(d) Neither Artinian nor Noetherian

15. *Let T be a linear transformation on* $\mathbb{C}^5$, *whose minimal polynomial is* $f(x) = (x-1)^2(x-2)(x-3)^2$.
Which of the following is the Jordan form for T?

(a) $\begin{bmatrix} 1&1&0&0&0 \\ 0&1&0&0&0 \\ 0&0&2&0&0 \\ 0&0&0&3&1 \\ 0&0&0&0&3 \end{bmatrix}$

(b) $\begin{bmatrix} 0&1&0&0&0 \\ 1&2&0&0&0 \\ 0&0&3&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&3 \end{bmatrix}$

(c) $\begin{bmatrix} 0&1&0&0&0 \\ 1&0&0&0&0 \\ 0&0&2&0&0 \\ 0&0&0&3&0 \\ 0&0&0&0&3 \end{bmatrix}$

(d) $\begin{bmatrix} 0&1&0&0&0 \\ 1&1&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&3&1 \\ 0&0&0&0&3 \end{bmatrix}$

16. Let $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ $(x\ real, n = 1, 2, 3 \ldots)$. Then $\lim_{n\to\infty} f_n'(0)$ is

(a) 1
(b) -1
(c) 0
(d) ∞

17. Which of the following is known as the Stirling's formula?

(a) $\lim_{x\to\infty} \frac{\Gamma(x+1)}{(x/e)^x\sqrt{2\pi x}} = 1$

(b) $\lim_{x\to\infty} \frac{\Gamma(x+1)}{(xe)^x\sqrt{2\pi x}} = 1$

(c) $\lim_{x\to\infty} \frac{\Gamma(x+1)}{(e^x)^x\sqrt{\frac{2\pi}{x}}} = 1$

(d) $\lim_{x\to\infty} \frac{\Gamma(x)}{(e^x/x)^x\sqrt{2\pi x}} = 1$

18. The value of $\int_0^2 \lfloor x \rfloor d(x^2)$ is

(a) $\frac{1}{3}$

(b) 4

(c) -3

(d) 3

19. The radius of convergence of the power series $\sum_{n=0}^{\infty}(4n^2 - n^3 + 3)z^n$ is

(a) 0

(b) 1

(c) 5

(d) ∞

20. The value of $\lim_{x\to 0}\frac{b^x-1}{x}$, $b > 0$ is

(a) 0

(b) ∞

(c) $\log b$

(d) 1

21. Let $f(x, y) = kxy - x^3y - xy^3$ for $(x, y) \in \mathbb{R}^2$, where k is a real constant. The directional derivative of f at the point (1, 2) in the direction of unit vector $u = (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ is $\frac{15}{\sqrt{2}}$. Then the value of k is

(a) 2

(b) 4

(c) 1

(d) -2

22. Which of the following statements is not true?

(a) Any set with outer measure different from zero is uncountable.

(b) Cantor set is an uncountable set with outer measure zero.

(c) The set of algebraic numbers is not countable.

(d) The set of rational numbers is countable as well as measurable.

23. If x *and* y are real numbers in [0, 1), then the sum modulus 1, $\dot{+}$ of x *and* y is defined by

$$x \dot{+} y = \begin{cases} x+y, & x+y<1 \\ x+y-1, & x+y\geq 1 \end{cases}$$

The operation $\dot{+}$ is

(a) commutative but not associative

(b) commutative and associative

(c) associative but not commutative

(d) neither commutative nor associative.

24. Which of the following is not true?

(a) Every step function need not be a simple function.

(b) A continuous function defined on a measurable set is measurable.

(c) If f is a measurable function, then so are $|f|, |f|^p$ $(p > 0)$, $\exp(cf)$, f^+ *and* f^-.

(d) A bounded function is Riemann integrable if and only if it is continuous almost everywhere.

25. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = |x|$. Then

(a) $D^+f(0) = D_+f(0) = -1$

(b) $D_-f(0) = D^-f(0) = 1$

(c) $D^+f(0) = D_+f(0) = D_-f(0) = D^-f(0) = 1$

(d) $D^+f(0) = D_+f(0) = 1$

26. Which of the following is not true?

(a) A function of bounded variation need not be continuous.

(b) A continuous function need not be of bounded variation.

(c) A function of bounded variation need not be measurable.

(d) A function of bounded variation is bounded but not conversely.

27. If $f(x) = |x|$ is a periodic function of period 2π, then the Fourier series for $f(x)$ is

(a) $f(x) = \frac{\pi}{2} - \frac{4}{\pi}[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots]$

(b) $f(x) = \frac{\pi}{2} - \frac{4}{\pi}[\frac{\sin x}{1^2} + \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} + \cdots]$

(c) $f(x) = \frac{\pi}{2} - \frac{4}{\pi}[\frac{\cos x}{1^2} - \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} - \cdots]$

(d) $f(x) = \frac{\pi}{2} - \frac{4}{\pi}[\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - \cdots]$

28. Which of the following statements is not true?

(a) The interval $(-\infty, a]$ is measurable for every real a.

(b) Every Borel set is measurable.

(c) Lebesgue measure is invariant under translation modulo 1.

(d) none of the above.

29. Which of the following statements is not true?

(a) Every measurable function is nearly continuous.

(b) Every convergent sequence of measurable functions is nearly uniformly convergent.

(c) Every measurable set is nearly a finite union of intervals.

(d) none of the above.

30. If $x = r\sin\theta \cos\varphi, y = r\sin\theta \sin\varphi, z = r\cos\theta$, then $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} =$

(a) $r\sin\theta$

(b) $r\cos\theta$

(c) $r^2\cos\theta$

(d) $r^2\sin\theta$

31. Consider the power series $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = \begin{cases} \frac{1}{3^n} \text{ if } n \text{ is even} \\ \frac{1}{5^n} \text{ if } n \text{ is odd} \end{cases}$

The radius of convergence of the series is equal to

(a) 1

(b) 3

(c) 5

(d) ∞

32. The maximum modulus of e^{z^2} on the

$$set\ S = \{z \in \mathbb{C} : 0 \leq Re(z) \leq 1, 0 \leq Im(z) \leq 1\}\ is$$

(a) $\frac{2}{e}$

(b) e

(c) $e + 1$

(d) e^2

33. Let $C = \{z \in \mathbb{C}: |z - i| = 2\}$. Then $\frac{1}{2\pi} \int_C \frac{z^2-4}{z^2+4} dz$ is equal to

(a) 2

(b) 1

(c) -2

(d) -1

34. A bounded entire function is constant. This statement is of

(a) Cauchy's theorem

(b) Liouville's theorem

(c) Morera theorem

(d) Schwarz theorem

35. The principal value of $(-1)^{\frac{-2i}{\pi}}$ is

(a) e^2

(b) e^{2i}

(c) e^{-2i}

(d) e^{-2}

36. Let $R = \{z = x + iy \in \mathbb{C} : 0 < x < 1 \text{ and } -11\pi < y < 11\pi\}$ and Γ be the positively oriented boundary of R. Then the value of the integral

$$\frac{1}{2\pi i} \int_{\Gamma} \frac{e^z}{e^z - 2} dz \text{ is equal to}$$

(a) 0

(b) 11
(c) 22
(d) 44

37. *The number of zeros of the polynomial*

$$2z^7 - 7z^5 + 2z^3 - z + 1 \text{ in the unit disc } \{z \in \mathbb{C} : |z| < 1\} \text{ is}$$

(a) 5
(b) 3
(c) 7
(d) 1

38. *If* $f(z) = \frac{z}{e^z - 1}$ *, then consider the following statements:*
S: $z = 0$ *is removable singularity.*
T: $z = 2n\pi i, n = \pm 1, \pm 2, \ldots$ are simple poles.
(a) *Both S and T are true*
(b) *Both S and T are false*
(c) *S is true and T is false*
(d) *S is false and T is true*

39. *Let* $f: \mathbb{C} \to \mathbb{C}$ *be non − zero and analytic function at all points in* $\mathbb{Z}$.
If $F(z) = \pi f(z) \cot(\pi z)\ \forall z \in \mathbb{C}\backslash\mathbb{Z}$, *then the residue of F at* $n \in \mathbb{Z}$ *is*
(a) $\pi f(n)$
(b) $f(n)$
(c) $\frac{f(n)}{\pi}$
(d) π

40. Consider the ordinary differential equation on $\mathbb{R}$, $y'(x) = f(y(x))$. If f is an even function and y is an odd function, then
(a) -y(-x) is also a solution
(b) y(-x) is also a solution
(c) -y(x) is also a solution
(d) y(x)y(-x) is also a solution.

41. Let $Y_1(x)$ and $Y_2(x)$ be two solutions of

$$Y''(x) + Y'(x) + Y(x) = 0 \text{ on } [0, 1].$$

Let W(x) be the Wronskian of Y_1 and Y_2 such that $W\left(\frac{1}{2}\right) = 0$.
Then which of the following holds?
(a) W(x) = 0 for all x∈ [0, 1]
(b) W(x) ≠ 0 for all x∈ [0, 1/2) ∪ (1/2, 1]
(c) W(x) > 0 for all x∈ (1/2, 1]
(d) W(x) < 0 for all x∈ [0, 1/2).

42. Let $y(t) = \binom{y_1(t)}{y_2(t)}$ satisfy $\frac{dy}{dt} = Ay; t > 0,\ y(0) = \binom{0}{1}$, where A is 2× 2 constant matrix with real entries satisfying trace(A) = 0 and det(A) > 0. Then $y_1(t)$ and $y_2(t)$ both are

(a) Monotonically decreasing functions of t on (0, 10π)

(b) Monotonically increasing functions of t on (0, 10π)

(c) Oscillating function of t on (0, 10π)

(d) Constant function of t on (0, 10π).

43. The critical point of the system

$\frac{dx}{dt} = -4x - y,\ \frac{dy}{dt} = x - 2y$ is an

(a) Asymptotically stable node

(b) Unstable node

(c) Stable spiral point

(d) Unstable saddle point.

44. If Λ is the set of all characteristics values of the Sturn-Liouville boundary value problem $y'' + \lambda y = 0, \lambda > 0, y(0)\ \&\ y(\pi) = 0$, then

(a) Λ is a finite set

(b) Λ is a countably infinite set

(c) Λ is an uncountable set

(d) Λ is an empty set.

45. Given a system $x'(t) = A\,x(t),\ A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 4 & -2 \end{bmatrix}$. If Φ(t) is its

fundamental matrix such that Φ (0)= $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then det(Φ(t)) is

(a) $t + 1$

(b) 1

(c) $t^2 + 1$

(d) $t^3 + 1$.

46. If $\phi_p(t)$ denotes the p^{th} approximate solution of initial value problem, $x'(t) = 1 + tx,\ x(0) = 1$, by the method of successive approximation. Then which of the following is correct?

(a) $\phi_2(t) = 1 + t + \frac{t^2}{2} + \frac{t^3}{3}$

(b) $\phi_2(t) = 1 + t + \frac{t^2}{2} + \frac{t^3}{3} + \frac{t^4}{8}$

(c) $\phi_1(t) = 1 + \frac{t^2}{2}$

(d) $\phi_2(t) = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!}$.

47. The adjoint equation of differential equation $t^2 \frac{d^2x}{dt^2} + 7t\frac{dx}{dt} + 8x = 0$, is

(a) $t^2x'' + 7tx' + 8 = 0$
(b) $t^2x'' - 3t^2x' + 3x = 0$
(c) $t^2x'' + 3tx' + 3x = 0$
(d) $t^2x'' - 3tx' + 3x = 0$.

48. For the non-linear autonomous system:

$$\frac{dx}{dt} = ye^x, \qquad \frac{dy}{dt} = e^x - 1,$$

the critical point (0, 0) is:

(a) Saddle point
(b) Spiral point
(c) Node
(d) Centre.

49. Let W be the Wronskian of n linearly independent solutions of a n^{th} order homogeneous linear differential equation. Consider the following statements:

I: $W(x) = W(x_0)\, exp\left[-\int_{x_0}^{x} \frac{a_1(t)}{a_0(t)}\, dt\right]$, $\forall$ x$\in$ [a, b]
II: $W(x) = 0\ \forall\ x \in [a, b]$
Which of the following holds?
(a) I is true but II is false
(b) II is true but I is false
(c) Both I & II are true
(d) Both I & II are false.

50. If we follow the method of successive approximations for a non-linear initial value problem $x'(t) = f(t, x), x(\tau) = \xi; x, f, \xi \in \mathbb{R}^n; t, \tau \in I = [a, b]$, then the error of approximation at the n^{th} stage is

(a) Unbounded
(b) Bounded
(c) Given conditions are insufficient to ascertain the boundedness of error
(d) None of these.

51. If $f \in C$ on a domain $D \subset (a, b) \otimes \mathbb{R}^n$ and ϕ is a solution of a system $x'(t) = f(t, x), x(\tau) = \xi$ on (a, b), then

(a) $\lim_{t\to 0} \phi(a + t)$ exists always
(b) $\lim_{t\to 0} \phi(a + t)$ does not exist
(c) $\lim_{t\to 0} \phi(a + t)$ exists if f is bounded on D
(d) None of these.

52. If G(t, s) is the Green's function of the boundary value problem

$$x''(t) = f(t), x(0) = x(1) = 0, \text{ then}$$

(a) $x(t) = \int_0^1 G(t,s)\, ds$

(b) $x(t) = -\int_0^1 G(t,s)\, f(s)\, ds$ and G(t, s)= $\begin{cases} t(1-s), t \le s \\ s(1-t), t \ge s \end{cases}$

(c) $x(t) = -\int_0^1 G(t,s)\, f(s)\, ds$ and G(t, s)= $\begin{cases} (1-t)s, t \le s \\ (1-s)t, t \ge s \end{cases}$

(d) $x(t) = -\int_0^1 G(t,s)\, ds$ and G(t,s)= $\begin{cases} ts & t \le s \\ (1-t)(1-s), t \ge s \end{cases}$

53. The differential equation

$$x'' - (t - sint)x = 0, \qquad t \ge 0$$

is

(a) oscillatory
(b) non-oscillatory
(c) oscillatory but not self-adjoint
(d) non-oscillatory and Euler's equation.

54. Consider the following statements:

I: If $\Phi(t)$ is a fundamental matrix of the system $x'(t) = A(t)x$, such that

$$A(t+w) = A(t), w \neq 0, -\infty < t < \infty,$$

then $\Phi(t+w) = \Phi(t), -\infty < t < \infty$.

II: If $\Phi(t)$ is a fundamental matrix of the system $x'(t) = A(t)x$, such that

$$A(t+w) = A(t), w \neq 0, -\infty < t < \infty,$$

then $\Phi(t+w) = \Phi(t)C, -\infty < t < \infty$, C being a constant non-singular matrix.

Which of the following holds?

(a) I is correct but II is not correct
(b) Both I & II are correct
(c) I is incorrect but II is correct
(d) Both I & II are incorrect.

55. Two metrices d_1 and d_2 on a set are said to be equivalent if and only if

(a) they induce the same topology
(b) $d_1(x,y) = d_2(x,y)\ \forall x, y \in X$
(c) $|d_1(x,y) - d_2(x,y)| < \epsilon\ \forall \epsilon > 0$
(d) none of the above

56. Co-countable topology on a countable space is same as:

(a) usual topology
(b) discrete topology
(c) indiscrete topology
(d) co-finite topology

57. If a property of a topological space is carried by every subspace of the topological space, then the property is called:

(a) topological property
(b) homeomorphic property
(c) isomorphic property
(d) hereditary property

58. Pasting lemma is a theorem in topology related to:
(a) homeomorphism
(b) sequential continuity
(c) continuity
(d) open and closed maps

59. Compact subset of a T_2 space is
(a) connected
(b) bounded
(c) closed
(d) compact

60. A continuous map on a compact set
(a) does not exist
(b) is uniformly continuous
(c) is a homeomorphism
(d) is an isometry

61. Which of the following is not true?
(a) second countability $\Rightarrow$ Lindelofness
(b) second countability $\Rightarrow$ separability
(c) second countability is hereditary
(d) none of the above

62. A normal space
(a) is a regular space
(b) need not be regular space
(c) completely normal space
(d) Hausdorff space

63. Choose the correct statement:
(a) A second countable space is first countable.
(b) A first countable space is second countable.
(c) A connected space is compact.
(d) A compact space is connected.

64. Which of the following is not an executable statement?
(a) Assignment statement
(b) FORMAT statement
(c) IF(expression) THEN statement
(d) Write statement

65. Which of the following is invalid FORTRAN assignment statement?

(a) $a = 5.0$

(b) $y = 4.0^{*}x^{**}2 + 7.5$

(c) $b + c = a$

(d) $x = y$

66. Which of the following is not a character function in FORTRAN 90?

(a) ACHAR (i)

(b) CHAR (i)

(c) ICHR (c)

(d) ADJUSTR (string)

67. $\tan^{-1}\left(\frac{x}{y}\right)$ in FORTRAN 95 is correctly written as

(a) TAN(X/Y)

(b) ATAN(X/Y)

(c) ATAN2(X/Y)

(d) ATAN2(X, Y)

68. If array= (1 0 0 0 0 2 0 3), then consider the statement

WHERE (array /=0) b_array = 1.5 * array

Which of the following is correct?

(a) b_array= (1.5 0 0 0 0 0 0 4.5)

(b) b_array= (1.5 0 0 0 0 3.0 0 4.5)

(c) b_array= (1.5 0 0 0 0 2 0 3)

(d) b_array= (1 0 0 0 0 3.5 0 4.5)

69. Read the following portion of a FORTRAN 90 Program:

```
digit_1=n-(n/10)*10
n = n/10
digit_2=n-(n/10)*10
n =.n/10
digit_3=n-(n/10)*10
n = n/10
digit_4=n-(n/10)*10
n = n/10
digit_5=n
sum=digit_1+digit_2+digit_3+digit_4+digit_5
```

Which of the following option is correct?

(a) This program finds the sum of digits of any integer n

(b) This program finds only the digit of the integer n at the unit place

(c) This program finds the sum of digits of an integer n of 5 digits

(d) This program finds only the digits of the integer n from unit place to thousands place.

70. Consider the following table :

Do Statement	Number of Iterations
A1: DO I = 2, 12, 3	B1: 10
A2: DO I = 1, 10	B2: 5
A3: DO I = -2, -11, -2	B3: 13
A4: DO I = -10, 2	B4: 4

Which of the following match is correct?

(a) A2-B1, A1-B2, A3-B3, A4-B4

(b) A1-B2, A2-B3, A3-B4, A4-B1

(c) A1-B4, A2-B1, A4-B3, A3-B2

(d) A1-B4, A2-B1, A3-B3, A4-B2

71. Which of the following statements is correct?

(a) Logical .NOT. and .AND. operators have same precedence level

(b) Unary + or – operators have same precedence as binary + or – operators have

(c) Precedence of == operator is higher than < operator

(d) Precedence of * operator is lower than ** operator

72. Which of the following is correct?

(a) INT(ABS(x)+0.5) will correctly round off x = - 1.6 to the nearest integer

(b) MOD(32.5, 10) = 2

(c) AMOD(33.0, 16.5) = 2.0

(d) Output of the MOD(k, 10) and IABS(MOD(k, 10)) will be always same

73. The dual space of the space c_0 is

(a) l^1

(b) l^∞

(c) c

(d) c_0

74. The space l^p is an inner product space, with

(a) $p = 2$

(b) $p \neq 2$

(c) $p \geq 1$

(d) $p \leq 1$

75. In an inner product space, $x \perp y$ if and only if

(a) $||x + \alpha y|| \leq ||x||$ for all scalars α

(b) $||x + \alpha y|| \geq ||x||$ for all scalars α

(c) $||x + \alpha y|| > ||x||$ for all scalars α

(d) $||x + \alpha y|| = ||x||$ for all scalars α

76. Which of the following statements is not true:

(a) Any two norms on a finite dimensional space are equivalent.

(b) The discrete metric on a vector space $X \neq \{0\}$ can not be obtained from a norm.

(c) In a Banach space, every absolutely convergent series is convergent.

(d) Every separable Banach space has a Schauder basis.

77. Let $f: R^3 \to R$ be defined by $f(x) = \xi_1\alpha_1 + \xi_2\alpha_2 + \xi_3\alpha_3$,

where $a = (\alpha_j) \in R^3$ is fixed. Then

(a) f is not bounded

(b) f is bounded and $||f|| = 3$

(c) f is bounded and $||f|| = ||a||$

(d) none of the above.

78. Let (e_k) be any orthonormal sequence in an inner product space X. Then for any

$$x, y \in X, \qquad \sum_{k=1}^{\infty} | < x, e_k >< y, e_k > |$$

(a) $= ||x||\,||y||$

(b) $\geq ||x||\,||y||$

(c) $\leq ||x||\,||y||$

(d) $\leq ||x||^2||y||^2$

79. Which of the following is not a reflexive space?

(a) $l^p, 1 < p < +\infty$

(b) $L^p[a, b]$

(c) $C[a, b]$

(d) Any Hilbert space

80. Let $T: l^2 \to l^2$ be defined by

$(\xi_1, \xi_2, \xi_3, \dots) \mapsto (0,0,\xi_3, \xi_4 \dots)$.

Which of the following is not true?

(a) T is bounded

(b) T is self-adjoint

(c) T is positive

(d) none of the above

81. Which of the following statements is not true?

(a) If a normed space X is reflexive, then X' is reflexive.

(b) If the dual space X' of a normed space X is separable, then X itself is separable.

(c) The normed space X of all polynomials with norm defined by $||x|| = \max_j |\alpha_j|$ $(\alpha_0, \alpha_1, \dots$the coefficients of $x)$ is complete.

(d) There exist real valued continuous functions whose Fourier series diverge at a given point t_0.

82. The extremal of the functional $\int_{x_0}^{x_1}(x + y')y'dx$ is given by

(a) $y(x) = \frac{x^2}{4} + c_1x + c_2$

(b) $y(x) = -\frac{x^2}{4} + c_1x + c_2$

(c) $y(x) = 4x^2 + c_1x + c_2$

(d) $y(x) = x^2 + c_1\frac{x}{4} + c_2$

83. If $\boldsymbol{F}_i, \boldsymbol{R}_i, \boldsymbol{w}_i$, $\delta\boldsymbol{r}_i$ denotes the external force, resultant force, acceleration and virtual displacement of i^{th} particle respectively for a constraint system of N particles. Then, the general equation of dynamics is

(a) $\sum_{i=1}^{N}(\boldsymbol{F}_i - m_i\boldsymbol{w}_i)\delta\boldsymbol{r}_i = 0$

(b) $\sum_{i=1}^{N}(\boldsymbol{F}_i + \boldsymbol{R}_i)\delta\boldsymbol{r}_i = 0$

(c) $\sum_{i=1}^{N}(\boldsymbol{R}_i - m_i\boldsymbol{w}_i)\delta\boldsymbol{r}_i = 0$

(d) $\sum_{i=1}^{N}(\boldsymbol{F}_i + m_i\boldsymbol{w}_i)\delta\boldsymbol{r}_i = 0$

84. For the Hamilton function $\boldsymbol{H}(t, q_i, p_i)$, the Canonical equations are defined as

(a) $\frac{dq_i}{dp_i} = \frac{\partial H}{\partial t}, \frac{dp_i}{dq_i} = -\frac{\partial H}{\partial t}$

(b) $\frac{d\dot{p}_i}{dq_i} = -\frac{\partial H}{\partial t}, \frac{\partial H}{\partial t} = -\frac{dq_i}{dp_i}$

(c) $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$

(d) $\frac{\partial H}{\partial q_i} = \frac{\partial t}{\partial p_i}, \frac{\partial H}{\partial p_i} = \frac{\partial t}{\partial q_i}$

85. Let $\phi(t, q_i, p_i)$ and $\psi(t, q_i, p_i)$ be two arbitrary functions, then the Poisson bracket is

(a) $(\phi\psi) = \sum_{i=1}^{n}(\frac{\partial\phi}{\partial\psi}\frac{\partial q_i}{\partial p_i} - \frac{\partial\phi}{\partial q_i}\frac{\partial\psi}{\partial p_i})$

(b) $(\phi\psi) = \sum_{i=1}^{n}(\frac{\partial\psi}{\partial p_i}\frac{\partial\phi}{\partial q_i} + \frac{\partial\phi}{\partial p_i}\frac{\partial\psi}{\partial q_i})$

(c) $(\phi\psi) = \sum_{i=1}^{n}(\frac{\partial q_i}{\partial\phi}\frac{\partial p_i}{\partial\psi} - \frac{\partial p_i}{\partial\phi}\frac{\partial q_i}{\partial\psi})$

(d) $(\phi\psi) = \sum_{i=1}^{n}(\frac{\partial\phi}{\partial q_i}\frac{\partial\psi}{\partial p_i} - \frac{\partial\phi}{\partial p_i}\frac{\partial\psi}{\partial q_i})$

86. The transformation $\widetilde{q_i} = \alpha q_i, \widetilde{p_i} = \beta p_i, \alpha \neq 0, \beta \neq 0$ is canonical with

(a) $\widetilde{H} = \alpha\beta H$

(b) $\widetilde{H} = (\alpha - \beta)H$

(c) $\widetilde{H} = \frac{\alpha}{\beta}H$

(d) $\widetilde{H} = (\alpha + \beta)H$

87. If f and g are integrals of equations of motion and H is Hamilton function, then the Possion bracket (fg) satisfies the equation

(a) $\frac{\partial}{\partial t}(fg) + \left((Hg)f\right) = 0$

(b) $\frac{\partial}{\partial t}(fg) + \left((fg)H\right) = 0$

(c) $\frac{\partial}{\partial t}(fH) + ((fg)H) = 0$

(d) $\frac{\partial}{\partial t}(gH) + ((fg)H) = 0.$

88. The extremum of the functional $I[y(x)] = \int_0^{\frac{\pi}{2}}(y'^2 - y^2)dx$, with $y(0) = 0, y(\pi/2) = 1$

(a) cannot be achieved

(b) $can\ be\ achieved\ for\ y = a\ sinx + b\ cosx, a\ and\ b\ being\ arbitrary$

(c) $can\ be\ achived\ only\ for\ y = sinx$

(d) $canot\ be\ achieved\ for\ y = sinx\ but\ can\ be\ achieved\ for\ some\ other\ y$

89. A bead slides on a smooth rod which is rotating about one end in a vertical plane with uniform angular velocity ω. If g denotes the acceleration due to gravity, then the Lagrange's function L is

(a) $\frac{1}{2}m(\dot{r}^2 + \dot{r}^2\omega^2) - mgr\sin\omega t$

(b) $\frac{1}{2}m(\dot{r}^2 - r^2\omega^2) + mgr\sin\omega t$

(c) $\frac{1}{2}m(r^2 + \dot{r}^2\omega^2) - mgr\sin\omega t$

(d) $\frac{1}{2}m(\dot{r}^2 + r^2\omega^2) - mgr\sin\omega t$

90. A rigid body is in rotation about a stationary axis u. For the independent coordinate we take the angle of rotation ϕ. The generalized force $Q = L_u$, $T = \frac{1}{2}I_u\dot{\phi}^2$, I_u is the moment of inertia of the body about the axis of rotation. Then the Lagrange's equation

$\frac{d}{dt}\frac{\partial T}{\partial \dot{\phi}} - \frac{\partial T}{\partial \phi} = Q$ will take the form

(a) $I_u\ddot{\phi} = L_u$

(b) $I_u\dot{\phi} = Q$

(c) $I_u\phi = \dot{L}_u$

(d) $I_u\dot{\phi}^2 = L_u$

91. Which of the following is not correct?

(a) $\frac{\partial}{\partial t}(\phi\psi) = \left(\frac{\partial\phi}{\partial t}\psi\right) + \left(\phi\frac{\partial\psi}{\partial t}\right)$

(b) $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$

(c) $I = \oint[\sum_{i=1}^{n}\delta p_i\,\delta q_i + H\,\delta t]$

(d) $\frac{\partial S}{\partial t} + H\left(q_i, \frac{\partial S}{\partial q_i}\right) = 0$

92. The partial differential equation $y\frac{\partial^2 u}{\partial x^2} + x\frac{\partial^2 u}{\partial y^2} = 0$, is hyperbolic in

(a) The second and fourth quadrants

(b) The first and second quadrants

(c) The second and third quadrants

(d) The first and third quadrants.

93. A general solution of second order partial differential equation $4u_{xx} - u_{yy} = 0$, in terms of two twice differentiable functions f and g, is of the form

$u(x, y) =$

(a) f(x) + g(y)

(b) f(x+2y) + g(x-2y)

(c) f(x+4y) + g(x-4y)

(d) f(4x+y) + g(4x-y).

94. Let u(x, t) satisfy the initial boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2};\ x \in (0,1), t > 0$$
$$u(x,0) = \sin(\pi x);\ x \in [0,1]$$
$$u(0,t) = u(1,t) = 0, t > 0$$

Then for x∈ (0, 1), $u\left(x, \frac{1}{\pi^2}\right)$ is equal to

(a) $esin(\pi x)$

(b) $e^{-1}\sin(\pi x)$

(c) $\sin(\pi x)$

(d) $\sin(\pi x)\, e^{-\frac{1}{\pi^2}}$

95. Let $u(x,t)$ satisfy for $x \in \mathbb{R}, t > 0$

$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + 2\frac{\partial^2 u}{\partial x^2} = 0.$$

A solution of the form $u = e^{\iota x}\, v(t)$ with $v(0) = 0$ and $v'(0) = 1$

(a) is necessarily bounded on $\{(x,t): x > 0, t > 0\}$

(b) satisfies $|u(x,t)| > e^t$

(c) is oscillatory in x

(d) satisfies $u(x,0) = 1$.

96. If $u(x_1, x_2, \ldots, x_n)$ is a real valued harmonic function, then

(a) u is constant for all n

(b) u is constant only for $n = 2$ and $n = 3$

(c) u is constant if that is bounded

(d) u is never constant.

97. Consider the following statements:

I: $\phi(x) = \frac{-1}{2\pi}\log|x|, x \in \mathbb{R}^2, x \neq 0$ is a fundamental solution of Laplace's equation

II: $\phi(x) = \frac{1}{n(n-2)\alpha(n)} \frac{1}{|x|^{n-2}}, x \in \mathbb{R}^n, n \geq 3, x \neq 0$, where $\alpha(n)$ is the volume of unit ball in $\mathbb{R}^n$, is a fundamental solution of Laplace's equation.

Which of the following option is correct?

(a) I is true but II is false

(b) I is false but II is true

(c) Both I & II are true

(d) Both I & II are false.

98. Given a boundary value problem

$\Delta u + f = 0$ in U, u = g on ∂U, Where U is an open and bounded and ∂U is C^1, and if $u \in C^2(\bar{U})$, then which of the following option is correct?

(a) the energy functional $I[w] = \int_U \frac{1}{2}|Dw|^2 - wf\, dx$ remains unchanged by u

(b) u minimizes the energy $I[w]$

(c) u maximizes the energy $I[w]$

(d) all of the above options are correct.

99. Consider the Neumann problem:

$u_{xx} + u_{yy} = 0, 0 < x < \pi, -1 < y < 1,$

$u_x(0,y) = u_x(\pi, y) = 0, u_y(x,-1) = 0, u_y(x,1) = \alpha + \beta sinx.$

The problem admits solution for

(a) $\alpha = 0, \beta = 1$

(b) $\alpha = -1, \beta = \frac{\pi}{2}$

(c) $\alpha = 1, \beta = \frac{\pi}{2}$

(d) $\alpha = 1, \beta = -\pi$.

100. The solution of wave equation

$$u_{tt} - \Delta u = f \ in \ \mathbb{R} \times (0, \infty),$$
$$u = 0, u_t = 0 \ on \ \mathbb{R} \times \{t = 0\}$$

is

(a) $u(x,t) = \frac{1}{2} \int_0^t \int_{x-s}^{x+s} f(y, t-s) dy\, ds$

(b) $u(x,t) = \frac{1}{2} \int_0^t \int_0^s f(y, t-s) dy\, ds$

(c) $u(x,t) = \int_0^t \int_{x-s}^{x+s} f(y, t-s) dy\, ds$

(d) $u(x,t) = \frac{1}{2} \int_0^t \int_{x-s}^{x+s} f(y, s) dy\, ds$