

Roll No.

Total Pages : 4

MDE/D-23

4434

MATHEMATICAL PHYSICS

Paper-PHY-101

Time Allowed : 3 Hours]

[Maximum Marks : 60

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Show that each irreducible representation of an abelian group is one dimensional. 3
- (b) Show that if $F(k)$ be the Fourier transform of $f(x)$ then the Fourier transform of $f(\lambda x)$ is $\frac{1}{\lambda} F\left(\frac{k}{\lambda}\right)$ where λ is a constant. 3
- (c) Show that $J_2(x) = \frac{2}{x} J_1(x) - J_0(x)$. 3
- (d) If $u(x, y) = x^2 - y^2$ is the real part of an analytic function $f(z) = u(x, y) + iv(x, y)$, find $v(x, y)$. 3

UNIT-I

2. (a) Define the terms : 6
 - (i) Group.
 - (ii) Cyclic group.
 - (iii) Abelian group.Give examples of each.
- (b) Define conjugate elements in a group. What is a class? Find out classes of symmetry group of equilateral triangle. 6
3. Describe various steps involved in the construction of character table and hence construct the character table for symmetry group of rectangle. 12

UNIT-II

4. (a) Expand the function $f(x) = x^2$ for $0 < x < 2\pi$ in Fourier series. 6
- (b) Find the Fourier transform of : 6
$$f(x) = \begin{cases} 1-x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$
5. (a) Using the Laplace transform of derivatives, find the Laplace transform of $F(t) = t^2$. 6

- (b) Find the inverse Laplace transform of

$$f(s) = \frac{1}{s(s^2 + 1)^2}. \quad 6$$

UNIT-III

6. (a) Obtain explicit expression for $J_{\frac{1}{2}}(x)$ and $J_{-\frac{1}{2}}(x)$ in terms of trigonometric functions. 6

- (b) Explain the Orthogonality property of Legendre's polynomials. 6

7. (a) Starting with the Legendre differential equation derive the associated Legendre differential equation and hence explain associated Legendre polynomials. 6

- (b) Using generating function, obtain the expression for Laguerre polynomials of order n. 6

UNIT-IV

8. (a) State and prove Cauchy integral theorem. 6

- (b) Explain the Laurent's series expansion of a function of complex variables. 6

9. (a) With the help of contour integration prove that : 6

$$\int_0^{2\pi} \frac{d\theta}{1 - 2p\cos(\theta) + p^2} = \frac{2\pi}{1 - p^2} \quad \text{where } p \text{ is a real constant number such that } 0 < p < 1.$$

- (b) Using the method of contour integration, evaluate the integration :

$$I = \int_0^\infty \frac{\sin(x)}{x^2 + 4x + 5} dx. \quad 6$$