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CMDE/D-23 **4436**

QUANTUM MACHANICS–I

Paper–PHY–103

Time Allowed : 3 Hours] [Maximum Marks : 60

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Consider that the state of a free particle is described by the wave function :

$\psi(r,t) = \text{C} \exp\{\iota(k.r - \omega t)\}$. What can you say about uncertainty in position and linear momentum? Explain whether the linear momentum and the energy of the particle can be specified simultaneously. 3

(b) A particle in a harmonic potential is in a state described by the wave function : $\psi(x) = \frac{1}{\sqrt{5}}[u_0(x) + u_n(x)]$, with $\{u_n(x)\}$ being the set of energy eigenfunctions. If one measures energy of this particle, what is the expected outcome? 3

(c) Prove that the orbital angular momentum $\hat{\text{L}}$ operator satisfies the relation : $\hat{\text{L}} \times \hat{\text{L}} = \iota\hbar\hat{\text{L}}$. 3

(d) What are parahelium and orthohelium? Why parahelium cannot change into orthohelium or vice versa in an optical transition? 3

UNIT–I

2. (a) Show that if the potential energy function V(r) is changed everywhere by a constant, the time-independent wave functions are unchanged. What is the effect on the energy eigenvalues? 3

(b) Consider a beam of mono-energetic radiation quanta or electrons incident on a diaphragm with two ultra-narrow holes. Predict and draw the type of pattern on the detector screen according to the laws of classical as well as quantum mechanics. 4

(c) Solve the (linear) momentum eigenvalue equation to obtain momentum eigenvalues and eigenfunctions. Normalize these eigenfunctions over the cubical box as well as the entire space. 5

3. (a) Find the energy eigenvalues for a particle of charge q and mass m subjected to an external static and homogeneous magnetic field **B**. What is the corresponding classical result? 8

(b) Radial part of wave function for hydrogen atom is given by : $R_{n,l}(\rho) = \exp(-\rho/2)\rho^l \sum_i a_i \rho^i$, with $a_0 \neq 0$ and $\frac{a_{i+1}}{a_i} = \frac{i+l+1-n}{(i+1)(i+2l+2)}$. Use this result to obtain $R_{1,0}(\rho)$ and $R_{2,1}(\rho)$. 4

UNIT–II

4\. (a) Consider a system described by a time-independent Hamiltonian $\hat{H}$. Give details of the Heisenberg picture for studying the time evolution of the state of the system. Obtain specifically the Heisenberg equation of motion. 8

(b) Consider a system described by the Hamiltonian operator: 4

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{1}{2}K\hat{x}^2$$

Use the Heisenberg equation obtained in part (a) to prove that :

$$\frac{d}{dt}\langle \hat{x} \rangle = \frac{\langle \hat{p}_x \rangle}{m}; \frac{d}{dt}\langle \hat{p}_x \rangle = -K\langle \hat{x} \rangle .$$

5\. (a) Show that the Hamiltonian operator of a one-dimensional harmonic oscillator can be expressed as : $\hat{H} = \left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)\hbar\omega; \hat{a}$ and $\hat{a}^\dagger$, are the usual destruction and creation operators. Using operators $\hat{a}$ and $\hat{a}^\dagger$, prove that $\langle n|\hat{x}|n\rangle = \langle n|\hat{p}_x|n\rangle = 0$. 7

(b) Work out eigenfunctions of the position operator $\hat{r}$ in co-ordinate representation. Using these eigenfunctions, construct the Hamiltonian matrix in the co-ordinate representation. 5

UNIT–III

6\. (a) Develop the operator $\hat{L}_z$ in cartesian and spherical polar co-ordinate systems and find its eigenfunctions and eigenvalues. Prove that the eigenfunctions thus obtained satisfy the orthogonality relation : $\int_0^{2\pi} \Phi_{m'}^*(\phi)\Phi_m(\phi)d\phi = \delta_{mm'}$ Further, explain whether it is possible to determine the precise direction of orbital angular momentum vector. 4,2,2

(b) Suppose an attempt is made to determine the orbital angular momentum components L_x and L_y simultaneously. Will there be uncertainties involved? If so, calculate the product of uncertainties : $\Delta L_x \Delta L_y$. 4

7\. (a) What do you understand by spin angular momentum of an elementary particle? Construct the angular momentum matrices for operators $\hat{S}^2, \hat{S}_x, \hat{S}_y$, and $\hat{S}_z$ for the case $s = 1/2$ in the representation in which S_2 and S_z are diagonal. Show that the matrices thus obtained are hermitian and anticommute. 8

(b) Suppose that a spin-1/2 particle is in the state :

$$\chi = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

What are the probabilities of getting $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ if you measure S_z? Also, calculate $\langle S_z \rangle$. 4

UNIT–IV

8. (a) Explain the physical meaning of terms "identical particles" and "distinguishability and indistinguishability of identical particles". Construct physically admissible wave functions for a system of *N* identical particles if the particles involved are bosons and fermions, respectively. Show that the system (whether bosons or fermions) cannot change from a state of one symmetry to a state of another symmetry. 8

(b) Let us suppose that two non-interacting particles, each having mass *m*, are confined to a one-dimensional infinite square well of width L. The oneparticle energy eigenfunctions and eigenvalues are :

$$u_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right); E_n = n^2K, K = \frac{\pi^2\hbar^2}{2mL^2}$$

Find ground state energy of the system, when the particles are distinguishable, identical bosons, and identical fermions, respectively. 4

9. (a) Construct total wave function (i.e., by including spin) for the electronic system of ^{4}He atom by ignoring the e-e repulsion. Explain whether the ground state is singlet or triplet. 8

(b) Why it is so that for a system of identical particles the states described by wave functions of the type $\psi_S \pm \psi_A$ are not allowed; here ψ_S and ψ_A are symmetric and anti-symmetric wave functions, respectively. 4