(c) Minimize the following expansions.

$$xy\bar{z} + x\bar{y}\,\bar{z} + \bar{x}yz + \bar{x}\,\bar{y}\,\bar{z}$$

and draw the circuit diagram. **5**

Unit IV

8. Prove that the bipartite graph G = (V, E) with bipartition (V_1, V_2) has a complete matching from V_1 to V_2 if and only if $|N(A)| \geq |A|$ for all subsets A of V_1. **15**

9. (a) What are the necessary and sufficient conditions for Euler's circuits and paths ? **7.5**

(b) Write and explain Prim's algorithm for finding MST. **7.5**

Roll No. .... **Total Pages : 04**

CMCS/D-23 **24038**

DISCRETE MATHEMATICAL STRUCTURES

MS-20-14

(CBCS)

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all. Question No. **1** is compulsory. Attempt *four* more questions, selecting *one* question from each Unit. All questions carry equal marks.

(Compulsory Question)

2. (a) What is a minimum spanning tree of a connected weighted graph ?

(b) What are the various normal forms of Boolean expression ?

(c) State Inclusion-Exclusion principle.

(d) Determine whether the function f from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, and $f(d) = 3$ is one-to-one.

(e) What is the power set of the set $\{0, 1, 2\}$?

(f) State and prove DeMorgan's law. **6×2.5=15**

Unit I

2\. (a) Prove that the relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots\ldots\ldots$ **5**

(b) Let m be an integer with $m > 1$. Show that the relation $R = \{(a, b) \mid a \equiv b \pmod{m}\}$ is an equivalence relation on the set of integers. **5**

(c) Describe various set identities. **5**

3\. (a) Given A = { 1, 2, 3, 4}. Consider the following relation in A :

$R = \{(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)\}$

(i) Draw its directed graph.

(ii) Is R (a) reflexive (b) symmetric (c) Transitive, or (d) antisymmetric ?

(iii) Find $R^2 = R \circ R$. **7.5**

(b) Prove that : A function $f : A \to B$ is invertible if and only if both one-to-one and onto. **7.5**

Unit II

4\. (a) Show that $\neg (p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ is logically equivalent by developing a series of logical equivalences.

(b) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

5\. (a) Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there ? **5**

(b) State and prove Pigeonhole Principle. **5**

(c) State and prove Binomial theorem. **5**

Unit III

6\. (a) Let c_1 and c_2 be real numbers. Suppose that $r^2 - c_1 r - c^2 = 0$ has two distinct roots r_1 and r_2. Then the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if and only if $\alpha_n = \alpha_1 r_1^{\,n} + \alpha_2 r_2^{\,n}$ for $n = 0, 1, 2, \ldots$, where α_1 and α_2 are constants. **7.5**

(b) Use generating functions to find the number of ways to select r objects of n different kinds if we must select at least one object of each kind. **7.5**

7\. (a) Draw the Hasse diagram for the poset ({2, 4, 5, 10, 12, 20, 25}, |) and tell which elements are maximal, and which are minimal ? **5**

(b) Determine whether the posets ({1, 2, 3, 4, 5}, |) and ({1, 2, 4, 8, 16}, |) are lattices. **5**