7. (b) Consider the Markov chain with t.p.m. :

$$P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$$

Classify the states of Markov chain.

Unit IV

9. State and explain the postulates of Poisson Process and derive the distribution of the size of the population at time 't' under this process. Also prove its additive property.

10. Describe the simple birth and death processes. Also discuss the effect of immigration on birth and death processes.

Roll No. .... **Total Pages : 04**

CMDQ/D-23 6524

STOCHASTIC PROCESS

ST-303 & ST-304

Opt (ii)

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Distinguish between discrete and continuous state space of a stochastic process with the help of example.

(b) Obtain mean and variance of Geometric distribution with the help of probability generating function (p.g.f).

(c) Describe absorbing and reflecting barriers.

(d) Define an ergodic state of Markov chain.

(e) Illustrate Polya Process.

Unit I

2. (a) Define a stochastic process and give its classification on the basis of state space and time domain.

(b) Define probability generating function (p.g.f). Show that p.g.f. of the sum of two independent random variables equals the product of their p.g.f's. Also obtain p.g.f. of Poisson distribution.

3. (a) Define convolution of sequences. Show that binomial distribution is the convolution of 'n' independent Bernoulli distributions.

(b) Write a detailed note on the role of Partial Fraction expansion in order to obtain generating function.

Unit II

4. (a) Explain the concept of recurrent events. Illustrate with examples when recurrent events are called periodic, transient or persistent.

(b) Show that the generating function of sequences $\{u_n\}$and $\{f_n\}$ for recurrent event are related by the relation

$U(s) = \dfrac{1}{1-F(s)}$, where symbols have their usual meanings.

5. State the classical ruin problem. Find an expression for the expected duration of the game which is expected to be finite.

Unit III

6. (a) Define the following :

(i) Closed Set

(ii) Markov Chain

(iii) Order of Markov Chain

(b) Show that for an irreducible Markov chain, all states are of the same type.

7. (a) Let $\{X_n, n \geq 0\}$ be a Markov chain with state space $\{1, 2, 3\}$ and one step transition probability matrix P as :

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

With initial distribution $P[X_0 = i] = \frac{1}{3}$; $i = 1, 2, 3$

Compute :

(i) $P[X_3 = 3 \mid X_1 = 1]$

(ii) $P[X_2 = 1, X_0 = 3]$