9. Let $X = \begin{pmatrix} X^{(1)} \\ X^{(2)} \end{pmatrix}$ be a random vector with covariance matrix Σ. How are the rth canonical correlation between $X^{(1)}$ and $X^{(2)}$ and rth pair of canonical variates obtained ? Also find the maximum likelihoood estimates of the canonical correlations and variates. **15**

Roll No. .... **Total Pages : 04**

LMDQ/M-24 7511

MULTIVARIATE ANALYSIS

ST-401

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Let (x_1, x_2) have a bivariate normal distribution with p.d.f. :

$$f(x_1, x_2) = \frac{1}{2\pi} \exp\left\{-\frac{1}{2}(x_1 - 1)^2 + (x_2 - 2)^2\right\}$$

find the marginal desnsity of x_1.

(b) If $X \sim N(\mu, \Sigma)$, write down the M.L. estimate of μ and Σ. Are these estimates unbiased ?

(c) Define Partial correlation coefficient. What are its limits ?

(d) What is Fisher's discriminant function for two populations ?

(e) Define Canonical correlection. Give its uses. **5×3=15**

Unit I

2. Evaluate the constant $k(>0)$ so that :

$$k.\exp\left[-\frac{1}{2}(x-b)' \mathrm{A}(x-b)\right]$$

is a p.d.f., 'A' being a positive definite matrix and 'b' is a real vector. Also write down the significance of 'b' and 'A'. **15**

3. If $\mathrm{X} \sim \mathrm{N}(\mu, \Sigma)$, then :

(i) Obtain the distribution of Z = DX, where D is a $q \times p$ matrix of rank $q \leq p$. **8**

(ii) Show that :

$$\phi_x(t) = e^{i't\mu - \frac{1}{2}t'\Sigma t}.$$ **7**

Unit II

4. Let $\mathrm{X}_1, \mathrm{X}_2, \ldots\ldots \mathrm{X}_\mathrm{N}$ be a random sample from $\mathrm{N}(\mu, \Sigma)$. Then prove that sample mean $(\overline{\mathrm{X}})$ is distributed according to $\mathrm{N}\left(\mu, \frac{\Sigma}{\mathrm{N}}\right)$ and independently of $\hat{\Sigma}$. Also show that $\mathrm{N}\hat{\Sigma}$ is distributed as $\sum_{\alpha=1}^{\mathrm{N}-1} z_\alpha z_\alpha'$, where z_α is distributed according to $\mathrm{N}(0, \Sigma)$, independently of $z_\beta (\alpha \neq \beta)$. **15**

5. Define multiple correlation coefficient between X_1 and $\mathrm{X}_2, \ldots\ldots \mathrm{X}_p$ and show that it is non-negative. Find its distribution when the population multiple correlation is zero. **15**

Unit III

6. (a) Define (i) T^2-statistics (ii) D^2-statistic and discuss their uses in two sample testing of hypotheses problems. **10**

(b) Give the concept of multivariate Behren's Fisher problem. **5**

7. Develop the Baye's solution for classifying an observation into one of the two given populations and derive the procedure when the two populations are multivariate normal. **15**

Unit IV

8. (a) Define Wishart distribution. State and prove its additive property. **8**

(b) Define sample generalized variance and obtain its distribution. **7**