Unit III

6. (a) Define multinomial distribution. Discuss its marginal and conditional distributions when $X = (X_1, X_2, \ldots\ldots, X_k)$ is divided into two groups.
   (b) Discuss the classical situation of hypergeometric distribution. Obtain its probability mass function and probability generating function.

7. (a) If X is a non-negative integer values *r.v.* satisfying the memory less property :

$$P(X > t + s \mid X > t) = P(X > s - 1)$$

   Then show that X follows geometric distribution.
   (b) Define negative binomial distribution in its various forms. Obtain its mode.

Unit IV

8. (a) Define gamma distribution. If X and Y are two independent gamma variates with parameters (a, m) and (a, n), find the distributions of :

$$U = X + Y \text{ and } V = X/Y$$

   (b) Write the p.d.f. of *t*-statistics and show that it tends to normal distribution as its d.f. approaches to infinity.

9. Define Chi-square and F-statistics and derive their distributions. Develop the interrelations between F, t and χ^2 distributions.

Roll No. .... **Total Pages : 04**

CMDE/D-23 6017

STATISTICAL METHODS AND DISTRIBUTION THEORY

ST-102

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Differentiate between probability mass function and probability density function.
   (b) Define mathematical expectation with its important properties.
   (c) State and prove the additive property of Poisson distribution.
   (d) Define t statistic, write its p.d.f. and give its applications.
   (e) Define moment generating function with its important properties and utility.

Unit I

2. Define joint, marginal and conditional probability density functions. Two random variables X and Y have the following joint pdf :

$$f_{xy}(x, y) = 2 - x - y; 0 \le x, y \le 1$$
$$= 0 \quad ; \text{otherwise}$$

Find :

(i) Marginal pdf of X and Y.

(ii) Conditional density functions of X and Y.

(iii) Var.(X) and Var. (Y)

(iv) Covariance and Correlation between X and Y.

3. State and prove the Baye's theroem with its applications. The contents of urn 1, 2, 3 are as follow :

Urn I : one white, two black and three red balls

Urn II : two white, one black and one red ball.

Urn III : four white, five black and three red balls.

One urn is choosen at random and two balls are drawn. They happened to be white and red.

What is the probability that :

(i) They came either from urn-I or urn-II ?

(ii) They came either from urn-II or urn-III ?

Unit II

4. (a) State and prove Chebyshev's inequality with its utility.

(b) For geometric distribution $p(x) = 2^{-x}$; $x = 1, 2, 3, ...$ Prove that Chebyshev's inequality gives :

$$P\left[|x-2| \le 2\right] > \frac{1}{2}$$

while the actual probability is 15/16.

5. (a) What do you mean by Correlation ? Discuss at least three properties of coefficient of correlation. A sample of 20 observations (x_i, y_i) gave the following sums :

$$\sum x_i^2 = 40, \sum x_i y_i = 12, \sum y_i^2 = 18, \sum x_i = 8, \sum y_i = 0$$

Calculate the coefficient of correlation.

(b) What do you mean by regression ? Two variables have the regression lines :

$$8x - 10y + 66 = 0 \text{ and } 40x - 18y = 214$$

Find :

(i) The mean values $\overline{x}$ and $\overline{y}$.

(ii) The regression coefficients b_{xy} and b_{yx}

(iii) The correlation coefficient r.