Where $x_0 = a$ and $x_{2n} = b$. Hence evaluate

$$\int_1^2 \frac{dx}{x}$$

Also give estimate of errors for $n = 1$ and $n = 2$. (Given that $\log_e 2 = 0.69315$)

9. Apply Runge-Kutta method of fourth order to solve : **15**

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0)1 \text{ at } x = 0.2,\ 0.4$$

Roll No. ..... **Total Pages : 04**

CMDE/D-23 6018

LINEAR ALGEBRA AND NUMERICAL ANALYSIS

ST-103

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. The weightage of all the questions is same.

Compulsory Question

1. Short answer type questions : **2.5×6=15**

(i) Show that the set of vectors :

$u = (1, 2, -3)$, $v = (1, -3, 2)$, and $w = (2, -1, 5)$ of V_3 is linearly independent.

(ii) Prove that three points (x_1, y_1), (x_2, y_2) and (x_3, y_3) in a plane are collinear iff rank of the matrix

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$ is less than 3.

(iii) Obtain characteristic roots of matrix, $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.

(iv) Show that :

$$(1 + \Delta)(1 - \nabla) \equiv 1.$$

(v) Show that $E\nabla \equiv \nabla E \equiv \nabla$.

(vi) Write a concise note on Numerical Integration.

Unit I

2. If V(F) and W(F) are vector spaces and T : V $\rightarrow$ W is a linear transformation. Suppose V is of dimension n. Prove that Rank (T) + Nullity (T) = dim (V). **15**

3. Show that the set : **15**

$$B = \{\alpha + i\beta, \gamma + i\delta\}$$

is a basis set of complex numbers,

$$C(\mathbb{R}) \quad iff \quad \alpha\delta \neq \beta\gamma$$

Unit II

4. Prove that every finite dimensional inner Product Space has an orthogonal basis. **15**

5. Reduce the quadratic form : **15**

$$2x_1x_2 + 2x_1x_3 - 2x_2^2 + 4x_2x_3 - x_3^2$$

to diagonal form. Also find equations of transformation.

Unit III

6. What is interpolation ? Discuss Newton's forward interpolation formulae in detail. How is it different from Newton's backward interpolation formula ? **15**

7. Solve the following system of equations using Gauss-Seidal method : **15**

$$5x_1 + 2x_2 + x_3 = 12$$

$$x_1 + 4x_2 + 2x_3 = 15$$

$$x_1 + 2x_2 + 5x_3 = 20$$

correct upto three decimal places.

Unit IV

8. Prove by Simpson's 1/3 formula : **15**

$$\int_a^b f(x)dx = \frac{b-a}{6n}[f(x_0) + 4f(x_1) + 2f(x_2) + \ldots\ldots\ldots\ldots + f(x_{2n})]$$