(b) State Inversion theorem and apply inversion formula corresponding to which the characteristic function is : **7**

$$\phi_x(t) = \begin{cases} 1-|t| & if \quad |t| \le 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

9. (a) State and prove De Moivre-Laplace Central limit theorem. **8**

(b) Let $\{X_n\}$ be a sequence of independent random variables with : **7**

$$P[X_n = +n] = \frac{n^{-\lambda}}{2} = P[X_n = -n]$$

$$P[X_n = 0] = 1 - n^{-\lambda}, \lambda > 0$$

Show that Central limit theorem holds for the sequence $\{X_n\}$ when $0 < \lambda < 1$.

Roll No. .... **Total Pages : 04**

CMDE/D-23 **6016**

MEASURE AND PROBABILITY THEORY
ST-101

Time : Three Hours] [Maximum Marks : 75

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define sigma field. Also give an example of sigma field.

(b) Explain the concept of Lebesque measure.

(c) State Dominated convergence theorem.

(d) Does convergence in probability imply convergence in distribution ? Explain briefly the answer.

(e) Discuss uniqueness property of characteristic function. **3×5=15**

Unit I

2. (a) Define lim A_n and lim A_n for an arbitrary sequence $\{A_n\}$. When the limit of a sequence of sets exists ? Explain. **8**

(b) Show that every sigma field is a field but the converse is not true. **7**

3. (a) Define measurable set. Prove that the intersection and difference of two measurable sets are also measurable. **7**

(b) Define : **8**

(i) Probability measure

(ii) Inner and outer measure.

(iii) Lebesque-Stieltjes measure.

Unit II

4. (a) Define a measurable function. If F is measurable function then show that | F | is also measurable. **5**

(b) Suppose $\{F_n\}$; $n = 1, 2, \ldots$ is a sequence of measurable functions such that $F_n : -0 \rightarrow R$. Show that $\overline{\lim}\, F_n$ and $\underline{\lim}\, F_n$ are measurable functions. **10**

5. (a) State and prove monotone convergence theorem by proving Fatou's Lemma. **10**

(b) Define convergence in measure and fundamental in measure. **5**

Unit III

6. (a) State and prove Borel-Cantellí Lemma. **8**

(b) Let X is a random variable having probability density function : **7**

$$f(x) = \begin{cases} e^{-x} & ; \quad 0 \le x < \infty \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Use Chebyshev's inequality to obtain lower bound to the probability $P[-1 < x < 3]$.

7. (a) Define : **10**

(i) Almost sure convergence

(ii) Convergence in rth mean.

Also show that convergence almost sure implies convergence in probability but converse is not true.

(b) Write a short note on Kolmogorov's inequality. **5**

Unit IV

8. (a) What are the laws of large numbers ? Discuss Bernoulli's form of the law of large numbers. **8**