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CMDQ/M-24 **5533**

LINEAR PROGRAMMING

Paper–MMATH 21-415

Time Allowed : 3 Hours] [Maximum Marks : 80

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. Write short notes on the following :

(i) Find the optimal value of the problem

Min $z = x_1 - 2x_2 + 3x_3$, subject to $2 \le x_1 \le 3$, $0 \le x_2 \le 2$, $-1 \le x_3 \le 1$.

(ii) If the optimal basis of linear programing problem Max $z = x_1 + 5x_2 + 3x_3$ subject to $x_1 + 2x_2 + x_3 = 3$, $2x_1 - x_2 = 4$, $x_1, x_2, x_3 \ge 0$ is $(x_1, x_3)^T$, then find the optimal value of the dual of the problem ?

(iii) Find the optimal solution of the problem

Minimize $z = 3x_1 + 2x_2$, subject to $3x_1 + 2x_2 \le 12$, $x_1 + 2x_2 \le 10$, $x_1, x_2 \ge 0$.

(iv) What can say about the dual of a LPP if the primal problem of the LPP is infeasible? Justify.

(v) What will be the maximum number of basic feasible solutions of a balanced Transportation problem having 5 sources and 3 destinations?

(vi) What can you say about the value of x_3 for which $(x_1 = 1, x_2 = 1, x_3)$ will be a basic feasible solution to the LPP with constraints: $x_1 + x_2 + 2x_3 = 4$, $2x_1 - x_2 + x_3 = 2$?

(vii) Show that the feasible region of an LPP is always a convex set.

(viii) Discuss the case when we have unbounded solution of an LPP in simplex method? Give proper reasoning in support of your answer.

UNIT–I

2. (a) Define basic feasible solution. Find all basic feasible solutions and hence optimal solutions for the problem:

Maximize $z = 3x_1 + 2x_2 - x_4$

subject to $x_1 + 2x_2 + 2x_3 = 4$,

$3x_1 - x_2 + 6x_3 + x_4 = 5$, $x_1, x_2, x_3, x_4 \ge 0$.

(b) Find all the extreme points of the set

$$S = \{(x_1, x_2) \mid x_1 + 2x_2 \geq -2, -x_1 + x_2 \leq 4, x_1 \leq 4\}$$

and represent the point (2, 3) as the convex linear combination of the extreme points of S.

3. (a) Use the simplex method to solve the following LPP:

Max $z = 4x_1 + 3x_2 + 2x_3 + x_4$

subject to $x_1 + 2x_2 + 2x_3 + 3x_4 \leq 12,$

$2x_1 + x_2 + 3x_3 + 2x_4 \leq 12,$

$x_1, x_2, x_3, x_4 \geq 0.$

Is the optimal solution unique? Why or why not?

(b) Prove or disprove that the sets

$$S_1 = \{(x_1, x_2, x_3) \in R^3 : 2x_1 - x_2 + x_3 \leq 6, x_1, x_2, x_3 \geq 0\}$$

and $S_2 = \{(x_1, x_2) \in R^2 : 0 < x_1^2 + x_2^2 < 1\}$

are convex sets?

UNIT–II

4. Solve the following LPP by Two phase method:

Max $z = 5x_1 - 3x_2 + 4x_3,$

Subject to $x_1 - x_2 \leq 1,$

$-3x_1 + 2x_2 + 2x_3 \leq 1,$

$4x_1 - x_3 = 1,$

x_1 unrestricted, $x_2 \geq 0,\ x_3 \geq 0.$

5. (a) For the general LPP: Max $Z = c^T x$ subject to $Ax = b, x \geq 0,$ let rank of the matrix A is m and M be the maximum absolute value of any entry of A. Then, prove that for every $\in > 0,$ such that

$$\in < \in_0 = \frac{1}{(m!)^2 M^{2m-1}}$$

the problem: Max $Z = c^T x$ subject to $Ax = b + \in,\ x \geq 0,$ is primal non-degenerate.

(b) Solve the following LPP using simplex method by applying the method to remove degeneracy:

Max $Z = 3x_1 + 9x_2$

subject to $x_1 + 4x_2 \leq 8,$

$x_1 + 2x_2 \leq 4,$

$x_1, x_2 \geq 0.$

UNIT–III

6. Solve the following problem by the revised simplex method:

   Max $z = 6x_1 - 2x_2 + 3x_3$,

   subject to $2x_1 - x_2 + 2x_3 \le 2$,

   $x_1 + 4x_3 \le 4$,

   $x_1, x_2, x_3 \ge 0.$

   State why is revised simplex method better than simplex method computationally?

7. Apply revised simplex method to compute the optimal solution of the following LPP:

   Min $z = 2x_1 + x_2$,

   subject to $3x_1 + x_2 = 3$,

   $4x_1 + 3x_2 \le 6$,

   $x_1 + 2x_2 \le 3$,

   $x_1, x_2 \ge 0.$

UNIT–IV

8. (a) Apply dual simplex method to solve the following LPP:

   Min $z = x_1 + 4x_2 + 3x_4$

   subject to $x_1 + 2x_2 - x_3 + x_4 \ge 3$,

   $-2x_1 + x_2 + 4x_3 + x_4 \ge 2$,

   $x_1, x_2, x_3, x_4 \ge 0.$

   Then, use complementary slackness theorem, to find the optimal solution of the dual of the above problem.

   (b) Write the dual of the following LPP:

   Min $z = 2x_1 + x_2 + x_3$

   subject to $x_1 + x_2 - x_3 \ge 1$,

   $-2x_1 + x_3 \le 0$,

   $x_1 - x_2 + x_3 = 2$,

   $x_1 \ge 0,\ x_2 \le 0,\ x_3$ unrestricted.

9. (a) It is given that the LPP: Max $z = p^T x$, s/t $Ax \le b,\ x \ge 0$ has an optimal solution. Suppose the requirement vector b is changed to another vector d. If the problem so obtained is feasible, then prove that it has an optimal solution.

(b) Solve the following transportation problem for minimum transportation cost :

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	20	19	14	21	16	40
O_2	15	20	13	19	16	60
O_3	18	15	18	20	–	70
Requirement	30	40	50	40	60	

where – indicates that it is not possible to transport goods from origin O_3 to destination D_5.