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CMDQ/M-24 **5530**

BOUNDARY VALUE PROBLEMS

Paper–MMATH 21-412

Time Allowed : 3 Hours] [Maximum Marks : 80

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **9** is compulsory. All questions carry equal marks.

UNIT–I

1. (a) Reduce the boundary value problem

$$y'' + \lambda y = 0,$$

$$y(0) = 0, \; y'(1) + v_2 y(1) = 0,$$

to a Fredholm integral equation. 8

(b) Let L be the differential operator,

$$Lu(s) = \left[A(s)\frac{d^2}{ds^2} + B(s)\frac{d}{ds} + C(s)\right]u(s),$$

$a < s < b$. Prove that,

$$\int (vLu - uMv)\,ds = \left[A(vu' - uv') + uv(B - A')\right]_a^b. \quad 8$$

2. (a) Find the Green's function for the equation

$$\frac{d^4y}{ds^4} + \lambda y = -f(s),$$

$$y(0) = 0 = y'(0), \;\; y(1) = 0 = y'(1). \quad 8$$

(b) Find the modified Green's function for the systems

$$y'' - \lambda y = 0, \;\; y(0) = y(1), \;\; y'(0) = y'(1). \quad 8$$

UNIT–II

3. Show that the Electro-static potential due to a thin circular disk is given by

$$f^n(\rho) = \int_b^a t\,\sigma^n(t) K_0(t, \rho)\,dt. \quad 16$$

4. (a) Given the Integral representation formula for the inhomogeneous equation :

$$\left(\nabla^2 - k^2\right)u = -4\pi\rho$$

with $u\big|_s = \tau, \; \left.\frac{\partial u}{\partial n}\right|_s = \sigma.$ 8

(b) Define Neumann problem and solve it. 8

UNIT–III

5. (a) Solve the integral equation :

$$f(s) = \int_0^s k\left(s^2 - t^2\right) g(t)\,dt, \; s > 0. \quad 8$$

(b) Define infinite Hilbert transform and solve the integral equation :

$$\sin s = \left(\frac{1}{\pi}\right)\int_{-\infty}^{*\infty}\left[\frac{g(t)}{(t-s)}\right]dt. \qquad 8$$

6. Solve the integral equation :

$$\int_0^a t\phi(t)\int_0^\infty J_1(P\rho)J_1(Pt)\,dp\,dt = \Omega\rho, \quad 0<\rho<a. \qquad 16$$

UNIT–IV

7. Discuss the approximate technique for solving the Fredholm integral equation of Ist kind

$$f(P) = \int_s K(P,Q)g(Q)\,dS, \; P \in S$$

and explain special cases of Kernel $K(P, Q)$. 16

8. (a) Solve the Boundary value problem :

$$\nabla_q^2 = \text{grad } P, \quad \text{div } q = 0$$

$q = e_1$, on S_1; $q_1 \to 0$ at ∞

for Steady stokes flow. 8

(b) Solve the problem of the diffraction of a plane wave by a soft sphere, take spherical polar coordinates. 8

Compulsory Question

9. Attempt all the following : 8×2=16

(i) Define Self adjoint initial value problem.

(ii) Give two properties of Green's function.

(iii) If $G(s;t) = \begin{cases} \left(\frac{s}{2t}\right)(1-t^2) & , \; s<t \\ \left(\frac{t}{2s}\right)(1-s^2) & , \; s>t \end{cases}$

then find the value of $\left(\frac{dG}{ds}\right)_{t+} - \left(\frac{dG}{ds}\right)_{t-} = ?.$

(iv) Write a short note on the existence of modified Green's function.

(v) Prove that : $G(P) = \frac{F(P)}{[1-K(P)]}.$

(vi) Solve : $\sin s = \int_0^s J_0(s-t)_g(t)\,dt.$

(vii) Find the Laplace transform of the function $f(s) = e^{as}.$

(viii) Give Green's second identity.