

ADVANCED DISCRETE MATHEMATICS

Paper-MMATH 21-409

Time allowed : 3 Hours]

[Maximum Marks : 80

Note : Attempt **five** questions in all, selecting **one** question from each unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. Attempt all questions:
 - (i) Describe universal bounds of a lattice.
 - (ii) State duality principle.
 - (iii) Define Switching circuit.
 - (iv) Write down the polynomials for subjunction and Pierce - operation.
 - (v) Describe Connected and Disconnected Graphs.
 - (vi) State Kuratowski's theorem.
 - (vii) Explain edge Connectivity and vertex Connectivity.
 - (viii) Define circuit matrix.

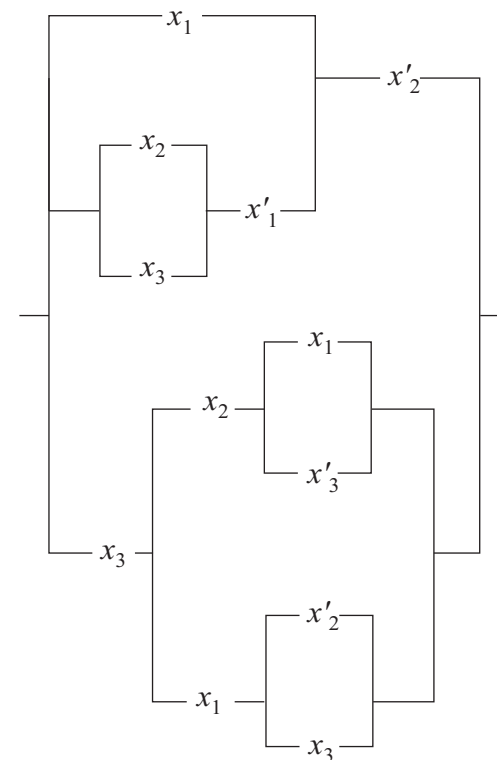
UNIT-I

2. (a) Show that any finite lattice is bounded. Find a lattice without a zero and a unit element.
- (b) Prove that every atom of a lattice with zero is join - irreducible.

3. (a) Prove that the cardinality of a finite Boolean algebra B is always of the form 2^n .
- (b) Prove that if B is a Boolean algebra, then the set $P_n(B)$ is a Boolean algebra and a sub-algebra of the Boolean algebra $F_n(B)$ of all functions from B_n to B.

UNIT-II

4. (a) Find the symbolic gate representation of the following diagram:



- (b) Explain NAND function and NOR function.

5. Describe the full adder circuit with the help of an example.

UNIT-III

6. (a) Prove that the number of vertices of odd degree in a graph is always even.
(b) Prove that if a graph (connected or disconnected) has exactly two vertices of odd degree, then there must be a path joining these two vertices.
7. (a) Prove that every tree has either one or two centers.
(b) Show that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G .

UNIT-IV

8. (a) Prove that the complete graph of five vertices is nonplanar.
(b) Prove that any simple planar graph can be embedded in a plane such that every edge is drawn as a straight-line segment.
9. Prove that the set of circuit vectors corresponding to the set of fundamental circuits, with respect to any spanning tree, forms a basis for the circuit subspace W_{Γ} .