Roll No. .... Total Pages : 4

CMDQ/M-24 **5522**

PARTIAL DIFFERENTIAL EQUATIONS

Paper–MMATH 21–402

Time Allowed : 3 Hours] [Maximum Marks : 80

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **9** is compulsory. All questions are carry equal marks.

UNIT–I

1. (a) State and prove strong maximum principle for Poisson equation. 8

   (b) State and prove Liouville's theorem. 8

2. (a) If $u \in \Omega$ is Harmonic, then prove that : 8

$$u(x) = \oint_{\partial B(x,r)} uds = \oint_{B(x,r)} udy$$

   for each ball $B(x,r) \subset \Omega$. Also prove the converse.

   (b) If u is Harmonic in Ω, then show that u is analytic in Ω. 8

UNIT–II

3. (a) Define Green's function and prove the symmetry property of Green's function. 8

   (b) Derive Green's function for a unit ball. 8

4. (a) Show that solution of the heat equation : 8

$$u_t - ku_{xx} = f(x,t), 0 \le x \le l, t > 0,$$

$$u(x,0) = \phi(x), u(0,t) = g(t), U(l,t) = h(t),$$

   where ϕ, g and h are smooth functions, is unique.

   (b) Solve the one-dimensional heat equation for an infinite rod with a heat source function $F(x,t)$ by making use of Duhamel's principle. 8

UNIT–III

5. Derive Kirchoff's and Poisson's formula for $n = 2$. 16

6. (a) Consider the wave equation in one-dimension. 8

$$u_{tt} - u_{xx} = 0 \text{ in } |R \times (0, \infty)$$

$$u = g, u_t = h, \text{ on } |R \times \{t = 0\},$$

   where g, h have compact supports. If the Kinetic and Potential energy are given by :

$$K(t) = 0.5 \int_{-\infty}^{\infty} u_t^2(x,t)dx, P(t) = 0.5 \int_{-\infty}^{\infty} u_x^2(x,t)dx$$

   respectively, then prove that $K(t) + P(t)$ is consant.

(b) State and prove the Uniqueness of solution for IVP for wave equation : 8

$u_{tt} - c^2 u_{xx} = f(x,t),\ -\infty < x < \infty,\ t > 0,\quad u(x,0) = \phi(x),$

$u_t(x,0) = \psi(x).$

UNIT–IV

7. (a) Find the solution $u(x, y)$ of the equation $u_x^2 + u_y^2 = 1$. 8

(b) State and derive the Hamilton Jacobi equation. 8

8. (a) Explain the separation of variables method for the wave equation : 8

$$u_{tt} - 4u_{xx} = 0,\ 0 \le x \le \pi,\ t > 0,$$

where the initial position and the initial velocity are given as 3 cos x and $1 - \cos(4x)$ respectively, and $u(0,t) = u(\pi,t) = 0$.

(b) Find the complete integral of $z^2 - pqxy = 0$ by Charpit's method where p and q denote the partial derivatives of z w.r.t. x and y respectively. 8

Compulsory Question

9. Attempt the following questions : 8×2=16

(a) State Harnack's inequality.

(b) Define Laplace and Fourier transforms.

(c) State mean value theorem for heat equation.

(d) State Dirichlet's principle.

(e) Define D'Alemberts formula for wave equation and write its one application for wave equation.

(f) Describe energy method for wave equation.

(g) Define transport equation for homogeneous and non-homogeneous systems.

(h) Define Legendre transform.