Dr. Richa Sharma
Deputy Director
SCD Division (In-charge)
Tel # 011-26742140
Email: specialcall@icssr.org

Indian Council of Social Science Research
(Ministry of Education)
JNU Institutional Area, Aruna Asaf Ali Marg
New Delhi – 110067
Website: www.icssr.org

SANCTION ORDER

F.No. 112/CRP-2023-2236/BBBP/SCD **Dated: 04-10-2023**

To,
The Registrar
Kurukshetra University,
Kurukshetra, Haryana, 136119

Subject: Sanction of **Short-term Empirical Research Project (Collaborative/Individual)** entitled **"The Impact Assessment of Beti Bachao Beti Padhao Scheme vis-à- vis Protection, Education and Participation of the Girl Child : A Multi-site study of select Districts of Haryana, Punjab and Rajasthan"** to **Dr. Amit Kumar, Kurukshetra University, Kurukshetra**

Dear Sir/Madam,

1. The Indian Council of Social Science Research (ICSSR) has approved the award of **"The Impact Assessment of Beti Bachao Beti Padhao Scheme vis-à- vis Protection, Education and Participation of the Girl Child : A Multi-site study of select Districts of Haryana, Punjab and Rajasthan"** the above Research Project submitted by **Dr. Amit Kumar** of your Institution.

2. The study, as proposed by the researcher, is to be located at and financially administered by your institution as per the guidelines of this award.

3. The ICSSR has sanctioned a grant-in-aid of **Rs. 1500000/-** for the above research project and the grant will be released as follows:

First instalment (50 % of sanctioned grant-in-aid)	**:**	**Rs. 750000/-**
Second instalment (25 % of sanctioned grant-in-aid)	**:**	**Rs. 375000/-**
Final instalment (25 % of sanctioned grant-in-aid)	**:**	**Rs. 375000/-**
Total	**:**	**Rs. 1500000/-**
Overhead charges over and above 7.5% or maximum Rs.1,00,000	**:**	**Rs. 100000/-**

4. The **First** installment of the approved grant-in-aid will be released after receiving the grant-in-aid bill duly filled in, stamped and signed by the Project Director as well as the affiliating organization**.**

5. As the study involves empirical research, the finalized schedules/questionnaires (2 copies) designed to elicit information should be sent to the ICSSR as per the following schedule:

   a) If the schedule /questionnaire for eliciting information is as per standard questionnaire, these will have to be sent to ICSSR immediately,

b) If the schedule /questionnaire for eliciting information are to be designed afresh keeping in view the requirements of the project, these will have to be sent to the ICSSR within a period of two months in any case.

6. The Second instalment will be released after receiving a satisfactory **three months** progress report, data summary report, along with a statement of expenditure for the first instalment and Grant-in-Aid Bill for second installment.

7. The Third and Final instalment will be released on receipt of (a) Satisfactory book length of the Final Report (Two Hard Copies plus one Soft Copy in Pen-Drive) in the publishable form after incorporating all corrections, suggestions of the expert; (b) soft and 2 hard copies of Executive Summary of Final Report; (c) Statement of accounts with Utilization Certificate in GFR of 12A form for the entire project amount duly signed by the Finance Officer/Registrar/Principal/Director of the affiliating institution (d) A certificate of statement of assets and books purchased out of the project fund issued by the affiliating institution. (e) The Grant-in-Aid Bill for the third installment.

8. Research undertaken by a Project Director may be reviewed by the Monitoring and Advisory Committee constituted by ICSSR and the project may be discontinued/ terminated, if research progress is found unsatisfactory or any ICSSR rules/guidelines are violated.

9. The Project Coordinator/Project Director would organize a workshop before submission of the final report. The workshop would deliberate on data collection process, compilation, organization and analyses of data on the respective scheme/policy initiative.

10. The ICSSR reserves all rights to publish the project funded by it, provided the work is recommended for publication by the ICSSR appointed expert/experts. In case, ICSSR approves the publication of the research work, the scholar should acknowledge that the project has been sponsored by the ICSSR, in all publications resulting from the project output (Research Paper, Books, Articles, Reports, etc.) and should submit a copy of the same to the ICSSR.

11. The Contingency Grant may be utilized for research and office assistance, books, stationary, computer cost, research assistance and the field work expenses of Project Coordinator, Co-Project Directors and research personnel connected with the research work.

12. The University/ Institution of affiliation will provide to the scholar office accommodation including furniture, library and research facilities and messenger services. For this, the ICSSR shall pay to the University/Institution of affiliation **overhead charges @7.5%** or maximum Rs. 1,00,000/- of the total expenditure incurred on the project only after successful completion of the project.

13. The accounts and the Utilization Certificate will be signed by the Finance Officer/Registrar/Principal/Director in the case of accounts of the institution are audited by CAG/AG. Otherwise, they need to be signed by the Finance Officer and the Chartered Account.

14. The Project Coordinator/ Project Director of the research project will be **Dr. Amit Kumar**, who will be responsible for the completion of the research project within **5/6 Months** from the date of commencement of the project, which is **15.09.2023** as intimated by the scholar.

15. In case, the Project Coordinator/Project Director fails to submit the periodic / final project report as per schedule with adequate justification, the scholar will be debarred from availing all financial assistance from ICSSR in future.

16. All grants from ICSSR are subject to the general provision of GFR 2017 and in particular with reference to the provision contained in GFR 209, GFR 210, GFR 211 and GFR 212.

17. The Project Coordinator/Project Director will ensure that the expenditure incurred by him conforms to the approved budget heads. The grant-in-aid is subject to all the conditions laid down in the **Indian Council of Social Science Research (ICSSR) Research Projects available in the ICSSR website www.icssr.org**

18. The expenditure on this account is debatable to the **Budget Head-ICSSR (Scheme Code 0877); OH 31.09 Research Projects**.

19. All instalments will be transferred through Public Finance Management System (PFMS) and ICSSR shall implement the EAT module for ensuring transparency of expenditure at all levels and to ensure that there is no parking of funds.

20. As per the instruction from MoE, the amount of grant sanctioned herein is to be utilized by **the end of the project duration.** The unspent amount shall be refunded to the ICSSR immediately on the expiry of the duration of the project. If the grantee fails to utilize the grant for the purpose for which the same has been sanctioned/or fails to submit the audited statement of expenditure within the stipulated period/ or fails to submit the final report within the stipulated time, the grantee will be required to refund the amount of the grant released with a penal interest thereon @ 10% per annum.

Yours faithfully,

Sd/- Dr. Richa Sharma
For MEMBER-SECRETARY

Encl: as above.

Copy to:

1. **Dr. Amit Kumar**
   Institute of Law,
   Kurukshetra University,
   Kurukshetra, Haryana 136119

2. Dr. Sunil Kumar,
   Assistant Professor,
   Deptt. Of Sociology,
   Kurukshetra University,
   Kurukshetra, Haryana-136119

3. Dr. Sadeep kumar Malyan
   Deptt of Environmental Studies,
   Dyal Singh Evening College,
   Lodi Road, University of Delhi,
   New Delhi-110003.

4. Dr. Sumit Kumar
   Institute of Law,
   Kurukshetra University,
   Kurukshetra, Haryana 136119

5. Finance Branch, ICSSR, New Delhi

6. Record file

Sd/- Dr. Richa Sharma
For MEMBER-SECRETARY

PROJECT BUDGET

Title: The Impact Assessment of Beti Bachao Beti Padhao Scheme vis-à- vis Protection, Education and Participation of the Girl Child : A Multi-site study of select Districts of Haryana, Punjab and Rajasthan

By: Dr. Amit Kumar

S. No.	Heads of Expenditure	Value
1.	Research Staff: Full time/Part-time/Hired Services	Not exceeding 40% of the total budget.
2.	Fieldwork: Travel/Logistics/Boarding, Survey Preparation or Consultancy etc.	Not exceeding 30%
3.	Workshop to disseminate the outcomes of the project	Up to 15% (not exceeding INR 2.00 lakh for collaborative research)
4.	Equipment and Study material: Computer, Printer, Source Material, Books, Journals, Software, Data Sets etc.	Not exceeding 10%
5.	Contingency	Not exceeding 5%
6.	Institutional Overheads (over and above the total cost of the project)	Affiliating Institutional overheads @ 7.5% of the approved budget , subject to a maximum limit of Rs 1,00,000/-

- **Remuneration and Emoluments of Project Staff**

  (a)Project staff could be engaged by the Project Coordinator/Project Director on a full/ part-time basis during the research work and the duration/consolidated monthly emoluments of their employment may be decided by the Project Coordinator/Project Director within the limits of the sanctioned financial allocation and as per the ICSSR rules.

  (b) Research Associate @ Rs.40,000/- p.m.. (Qualification – Post graduate in any social science discipline with minimum 55% marks and NET/SLET/M.Phil/Ph.D)

  (c) Research Assistant @ Rs.32,000/-p.m.(Qualification-Ph.D./M.Phil./ Post graduate in social science discipline with minimum 55%)

  (d) Field Investigator @ Rs.30,000/-p.m. (not exceeding 3 months) (Qualification- Post graduate in any social science discipline with minimum 55% marks)

  (e)Retrospective payment for work already done is not permissible.

- **Re-appropriation**: The Institution may re-appropriate expenditure from one head to another up to 10% of the sanctioned budget with the prior approval of the ICSSR.
- **Selection of Research Staff** should be done through an advertisement and a selection committee consisting of (1) Project Coordinator/ project Director; (2) One outside Expert (other than the Institute where the project is located); (4) Head of the Department/Dean of relevant faculty.
- **For all field work related expenses** of Project Coordinator/Project Director, Co-Project Director and project personnel, rules pertaining to affiliating institutes shall be followed.
- **All equipment and books** purchased out of the project fund shall be the property of the affiliating institutions. On completion of the study, the Project Coordinator/ Project Director shall submit an undertaking in this regard. The ICSSR, however, reserves the right to take charge of equipment and books, if it thinks it fit in a case.
- **Purchase of equipment/ assets** for the Research Project is permissible only if it is originally proposed and approved by the ICSSR and does not exceed the permissible amount.
- **No publication/presentation** in any form related the awarded research shall be made by the researcher or any member of the research team without prior approval of the ICSSR

Dr. Richa Sharma
Deputy Director
SCD Division (In-charge)
Tel # 011-26742140
Email: specialcall@icssr.org

Indian Council of Social Science Research
(Ministry of Education)
JNU Institutional Area, Aruna Asaf Ali Marg
New Delhi – 110067
Website: www.icssr.org

SANCTION ORDER

F.No. 91/CRP-2023-2290/SIM(PMKVY)/SCD **Dated: 26-09-2023**

To,
The Principal
Dyal Singh Evening College, Delhi
Delhi (NCT), 110003

Subject: Sanction of **Short-term Empirical Research Project (Collaborative/Individual)** entitled **"Evaluation of PMKVY: A Multi-site study of select Districts of Haryana, Western Uttar Pradesh, and Uttarakhand"** to **Dr. Sandeep Kumar Malyan, Dyal Singh Evening College, Delhi**

Dear Sir/Madam,

1. The Indian Council of Social Science Research (ICSSR) has approved the award of **"Evaluation of PMKVY: A Multi-site study of select Districts of Haryana, Western Uttar Pradesh, and Uttarakhand"** the above Research Project submitted by **Dr. Sandeep Kumar Malyan** of your Institution.

2. The study, as proposed by the researcher, is to be located at and financially administered by your institution as per the guidelines of this award.

3. The ICSSR has sanctioned a grant-in-aid of **Rs. 1300000/-** for the above research project and the grant will be released as follows:

First instalment (50 % of sanctioned grant-in-aid)		**Rs. 650000/-**
Second instalment (25 % of sanctioned grant-in-aid)		**Rs. 325000/-**
Final instalment (25 % of sanctioned grant-in-aid)		**Rs. 325000/-**
Total		**Rs. 1300000/-**
Overhead charges over and above 7.5% or maximum Rs.1,00,000	**:**	**Rs. 97500/-**

4. The **First** installment of the approved grant-in-aid will be released after receiving the grant-in-aid bill duly filled in, stamped and signed by the Project Director as well as the affiliating organization**.**

5. As the study involves empirical research, the finalized schedules/questionnaires (2 copies) designed to elicit information should be sent to the ICSSR as per the following schedule:

   a) If the schedule /questionnaire for eliciting information is as per standard questionnaire, these will have to be sent to ICSSR immediately,

   b) If the schedule /questionnaire for eliciting information are to be designed afresh keeping in view the requirements of the project, these will have to be sent to the ICSSR within a period of two months in any case.

6. The Second instalment will be released after receiving a satisfactory **three months** progress report, data summary report, along with a statement of expenditure for the first instalment and Grant-in-Aid Bill for second installment.

7. The Third and Final instalment will be released on receipt of (a) Satisfactory book length of the Final Report (Two Hard Copies plus one Soft Copy in Pen-Drive) in the publishable form after incorporating all corrections, suggestions of the expert; (b) soft and 2 hard copies of Executive Summary of Final Report; (c) Statement of accounts with Utilization Certificate in GFR of 12A form for the entire project amount duly signed by the Finance Officer/Registrar/Principal/Director of the affiliating institution (d) A certificate of statement of assets and books purchased out of the project fund issued by the affiliating institution. (e) The Grant-in-Aid Bill for the third installment.

8. Research undertaken by a Project Director may be reviewed by the Monitoring and Advisory Committee constituted by ICSSR and the project may be discontinued/ terminated, if research progress is found unsatisfactory or any ICSSR rules/guidelines are violated.

9. The Project Coordinator/Project Director would organize a workshop before submission of the final report. The workshop would deliberate on data collection process, compilation, organization and analyses of data on the respective scheme/policy initiative.

10. The ICSSR reserves all rights to publish the project funded by it, provided the work is recommended for publication by the ICSSR appointed expert/experts. In case, ICSSR approves the publication of the research work, the scholar should acknowledge that the project has been sponsored by the ICSSR, in all publications resulting from the project output (Research Paper, Books, Articles, Reports, etc.) and should submit a copy of the same to the ICSSR.

11. The Contingency Grant may be utilized for research and office assistance, books, stationary, computer cost, research assistance and the field work expenses of Project Coordinator, Co-Project Directors and research personnel connected with the research work.

12. The University/ Institution of affiliation will provide to the scholar office accommodation including furniture, library and research facilities and messenger services. For this, the ICSSR shall pay to the University/Institution of affiliation **overhead charges @7.5%** or maximum Rs. 1,00,000/- of the total expenditure incurred on the project only after successful completion of the project.

13. The accounts and the Utilization Certificate will be signed by the Finance Officer/Registrar/Principal/Director in the case of accounts of the institution are audited by CAG/AG. Otherwise, they need to be signed by the Finance Officer and the Chartered Account.

14. The Project Coordinator/ Project Director of the research project will be **Dr. Sandeep Kumar Malyan**, who will be responsible for the completion of the research project within **5/6 Months** from the date of commencement of the project, which is **15-09-2023** as intimated by the scholar.

15. In case, the Project Coordinator/Project Director fails to submit the periodic / final project report as per schedule with adequate justification, the scholar will be debarred from availing all financial assistance from ICSSR in future.

16. All grants from ICSSR are subject to the general provision of GFR 2017 and in particular with reference to the provision contained in GFR 209, GFR 210, GFR 211 and GFR 212.

17. The Project Coordinator/Project Director will ensure that the expenditure incurred by him conforms to the approved budget heads. The grant-in-aid is subject to all the conditions laid down in the **Indian Council of Social Science Research (ICSSR) Research Projects available in the ICSSR website www.icssr.org**

18. The expenditure on this account is debatable to the **Budget Head-ICSSR (Scheme Code 0877); OH 31.09 Research Projects**.

19. All instalments will be transferred through Public Finance Management System (PFMS) and ICSSR shall implement the EAT module for ensuring transparency of expenditure at all levels and to ensure that there is no parking of funds.

20. As per the instruction from MoE, the amount of grant sanctioned herein is to be utilized by **the end of the project duration.** The unspent amount shall be refunded to the ICSSR immediately on the expiry of the duration of the project. If the grantee fails to utilize the grant for the purpose for which the same has been sanctioned/or fails to submit the audited statement of expenditure within the stipulated period/ or fails to submit the final report within the stipulated time, the grantee will be required to refund the amount of the grant released with a penal interest thereon @ 10% per annum.

Yours faithfully,

(sd/- Dr. Richa Sharma)
For MEMBER-SECRETARY

Encl: as above.

Copy to:

1. **Dr. Sandeep Kumar Malyan**
   Department of Environmental Studies, Dyal Singh Evening College, Lodi Road , University of Delhi, New Delhi 110003
   Delhi (NCT) 110003
2. Dr. Sunil Kumar,

   Department of Sociology,

   Kurukshetra University Haryana, 7015155877
3. Dr. Amit Kumar, Institute of Law, Kurukshetra University, Kurukshetra (Haryana), 996510039
4. Prof. Vivek Kumar, CRDT,

   IIT DELHI. HAUZ KHAS,

   NEW DELHI - 110016

   9412619735 vivekk@iitd.ac.in
5. Finance Branch, ICSSR, New Delhi
6. Record file

(sd/- Dr. Richa Sharma)
For MEMBER-SECRETARY

PROJECT BUDGET

Title: Evaluation of PMKVY: A Multi-site study of select Districts of Haryana, Western Uttar Pradesh, and Uttarakhand

By: Dr. Sandeep Kumar Malyan

S. No.	Heads of Expenditure	Value
1	Research Staff: Full time/Part-time/Hired Services	Not exceeding 40% of the total budget.
2	Fieldwork: Travel/Logistics/Boarding, Survey Preparation or Consultancy etc.	Not exceeding 30%
3	Workshop to disseminate the outcomes of the project	Up to 15% (not exceeding INR 2.00 lakh for collaborative research)
4	Equipment and Study material: Computer, Printer, Source Material, Books, Journals, Software, Data Sets etc.	Not exceeding 10%
5	Contingency	Not exceeding 5%
	Institutional Overheads (over and above the total cost of the project)	Affiliating Institutional overheads @ 7.5% of the approved budget , subject to a maximum limit of Rs 1,00,000/-

- **Remuneration and Emoluments of Project Staff**

  (a)Project staff could be engaged by the Project Coordinator/Project Director on a full/ part-time basis during the research work and the duration/consolidated monthly emoluments of their employment may be decided by the Project Coordinator/Project Director within the limits of the sanctioned financial allocation and as per the ICSSR rules.

  (b) Research Associate @ Rs.40,000/- p.m.. (Qualification – Post graduate in any social science discipline with minimum 55% marks and NET/SLET/M.Phil/Ph.D)

  (c) Research Assistant @ Rs.32,000/-p.m.(Qualification-Ph.D./M.Phil./ Post graduate in social science discipline with minimum 55%)

  (d) Field Investigator @ Rs.30,000/-p.m. (not exceeding 3 months) (Qualification- Post graduate in any social science discipline with minimum 55% marks)

  (e)Retrospective payment for work already done is not permissible.

- **Re-appropriation**: The Institution may re-appropriate expenditure from one head to another up to 10% of the sanctioned budget with the prior approval of the ICSSR.
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- **For all field work related expenses** of Project Coordinator/Project Director, Co-Project Director and project personnel, rules pertaining to affiliating institutes shall be followed.

- **All equipment and books** purchased out of the project fund shall be the property of the affiliating institutions. On completion of the study, the Project Coordinator/ Project Director shall submit an undertaking in this regard. The ICSSR, however, reserves the right to take charge of equipment and books, if it thinks it fit in a case.
- **Purchase of equipment/ assets** for the Research Project is permissible only if it is originally proposed and approved by the ICSSR and does not exceed the permissible amount.
- **No publication/presentation** in any form related the awarded research shall be made by the researcher or any member of the research team without prior approval of the ICSSR

DEENBANDHU CHHOTU RAM UNIVERSITY OF SCIENCE & TECHNOLOGY
MURTHAL-SONEPAT-131039
HARYANA

Ref. DCRUST/R3/18/80

Dated 11-10-2018

To

The Chairman,
Department of Biochemistry
Kurukshetra University,
Kurukshetra-136119

Sub: Consent Letter to be a Co-Supervisor of PhD student

With reference to above mentioned subject, I, **Dr. Krishan Kumar Selwal**, Assistant Professor, Department of Biotechnology, DCRUST, Murthal hereby giving my consent to act as a Co-Supervisor of Mr. Sandeep Kumar who is pursuing his PhD research under the supervision of Dr. Vinita Bhankar, Assistant Professor, Department of Biochemistry, Kurukshetra University, Kurukshetra. I will guide him for entire duration of his research work and will supervise him throughout the research process.

Thanking you.

11/10/18

Dr. Krishan Kumar Selwal
Assistant professor

A copy of the above is forwarded to the following for information

1. PA to V.C. for kind information to Hon'ble Vice Chancellor
2. PA to Registrar for kind information to Registrar
3. Chairperson, Department of Biotechnology for information and record
4. Coordinator Research for information and record

Chairperson
Department of Biochemistry
Kurukshetra University
KURUKSHETRA-136119

Ph.: 0130-2484005, Fax : 0130-2484005, E-mail : registrar@dcrustm.org
website : www.dcrustm.ac.in, www.dcrustm.org

1/28/22, 12:55 PM

From:
Sonia Sindhu, Professor & Head,Vety. Physiology & Biochemistry

To,
J.B.Phogat, Dean PGS, Dean CODST,Dean PGS
Sandeep Kumar, Sr. Scientist,Vety. Physiology & Biochemistry

Document No.:- PB1323-140122

Subject: Permission for inclusion of name as Co-Major Advisor/Guide for carrying of PhD degree of Student fromKurukshetra University

Please find the attachment after doing the needfull.

Pl. send the case again as discussed telephonically.

J.B.Phogat
Dean PGS,Dean CODST
Post Graduate Studies
Jan 14 2022 10:26AM

All the required information has already been attached please.

Sonia Sindhu
Professor & Head
Department of Vety. Physiology and Biochemistry
Jan 14 2022 10:32AM

For n/a pl.

J.B.Phogat
Dean PGS,Dean CODST
Post Graduate Studies
Jan 14 2022 10:37AM

The request of Dr. Sandeep Kumar to act as Co-Major Advisor in the Advisory Committee of the PG student from KU, Kurukshetra may kindly be perused. The case for inclusion of Dr. Sandeep Kumar has discussed in the 270th meeting of DAC held on 13.01.2022 at 12:00 noon and same has been recommended with the following riders: 1. There will be no financial liability on the University. 2. The publications/IPRs arising out of the said work will be jointly shared by the two Universities with LUVAS teacher as corresponding author/co-author. Keeping in view of above, the inclusion of name of Dr. Sandeep Kumar, Sr. Scientist, VPB, LUVAS as a Co-Major Advisor/Guide in the Advisory Committee of PG student (Pooja whi, Roll No. 2112) of KUK may be allowed under Rule 5.5 (a) of Chapter III of University Calendar Volume II as LUVAS has MoU with KUK. Therefore, if agreed, may allow the inclusion of Dr. Sandeep Kumar in the advisory committee of concerned student, please.

Rakesh Kumar
Clerk
Post Graduate Studies
Jan 22 2022 3:23PM

Allowed as proposed above

J.B.Phogat
Dean PGS,Dean CODST
Post Graduate Studies
Jan 22 2022 4:02PM

For information and necessary action

Sonia Sindhu

Chairperson
Department of Biochemistry
Kurukshetra University
KURUKSHETRA-136119

"AN INTEGRATED STUDY ON WATER CONSUMPTION, WASTE MANAGEMENT AND OCCUPATIONAL HEALTH OF WORKERS IN A DELHI METRO RAIL CONSTRUCTION PROJECT – A CASE STUDY"

A

DISSERTATION REPORT

SUBMITTED IN PARTIAL FULFILLMENT FOR THE AWARD OF

DEGREE OF

MASTER OF TECHNOLOGY

IN

ENERGY & ENVIRONMENTAL MANAGEMENT

Submitted by

Name- Aman Bargoti

Roll no- 2022010002

EXTERNAL SUPERVISOR

Miss Manisha Kadyan

Chief Environment Officer

YFC Projects Pvt. Ltd. DMRC DC-04

INTERNAL SUPERVISOR

Dr. Sandeep Gupta

Assistant Professor

Institute of Environmental Studies,

Kurukhetra University, Kurukhetra

INSTITUTE OF ENVIRONMENTAL STUDIES,

KURUKSHETRA UNIVERSITY, KURUKSHETRA

(2022-2024)

Gmail

sandeep gupta <sandeep.gupta@kuk.ac.in>

FW: Recommendations of the NGP Expert Committee on project proposals under the CFP Geospatial Science Development

Ujjwal Rajput <ujjwal.rajput@gov.in>
To: blohani@iitk.ac.in
Cc: Shubha Pandey <shubha.p@nic.in>, dstgajendra2022@gmail.com, harshavardhandst@gmail.com, sandeep gupta <sandeep.gupta@kuk.ac.in>
Wed, Aug 14, 2024 at 9:09 AM

Dear Sir,

With reference to project no. NGP/GS-14/Bharat/IIT-K/UP/2023 titled "Development of Upscaling Model by Integrating Terrestrial LiDAR and Aerial LiDAR for Individual Tree Morphology, Above Ground Biomass, and Species Identification to Estimate Carbon Stock" sanctioned by NGP, DST, kindly note that Dr. Sandeep Gupta, Assistant Professor, Institute of Environmental studies, Kurukshetra University is the co-PI of the project.

Regards,

उज्ज्वल राजपूत / Ujjwal Rajput
वैज्ञानिक 'सी' / Scientist 'C'
राष्ट्रीय भूस्थानिक कार्यक्रम / National Geospatial Programme
विज्ञान और प्रौद्योगिकी विभाग / Department of Science and Technology
भारत सरकार / Government of India

Ph. +91 11 2659 0512

From: blohani@iitk.ac.in
To: "Shubha Pandey" <shubha.p@nic.in>
Cc: dstgajendra2022@gmail.com, harshavardhandst@gmail.com, "Ujjwal Rajput" <ujjwal.rajput@gov.in>, "sandeep gupta" <sandeep.gupta@kuk.ac.in>
Sent: Saturday, August 10, 2024 6:06:51 AM
[Quoted text hidden]
[Quoted text hidden]

ICAR-Central Soil Salinity Research Institute, Karnal (Haryana) -132001
भाकृअनुप.केन्द्रीय मृदा लवणता अनुसंधान संस्थान, करनाल (हरियाणा) -**132001**
Division of Soil and Crop Management
मृदा एवं फसल प्रबंध विभाग
Zarifa Farm, Kachhwa Road, Karnal-132001 (Haryana) India
जरीफा फार्म, काछवा रोड, करनाल–132001; हरियाणा इंडिया

Dr. Gajender

Dated: 02 April, 2024

Senior Scientist (Agronomy)
ICAR-CSSRI, Karnal-132001
Email: *gajender.icar@gmail.com*

To,
The Head,
Institute of Environment studies
Kurukshetra University, Kurukshetra

Subject: Consent for Co-Supervision of Ms. Pinki, PhD Research

Dear Sir/madam,

In reference to the ICAR-CSSRI office order F.No. 4(14)/ Estt. Sci. /2024/3945 dated 19.03.2024 in response to email from Dr. Dipti, I have been nominated as Co-supervisor of Ms. Pinki for her PhD research work at ICAR-CSSRI, Karnal. I hereby, convey my consent to serve as a co-supervisor for the above mentioned PhD research in collaboration with Kurukshetra University, Kurukshetra.

Under this Co-Supervision arrangement, we will provide guidance, support and utilize our knowledge and technical research support to enhance the quality of your research. We are looking forward to commencing this collaboration and contributing to achieve the research objectives.

Kind Regards

(Dr. Gajender)

भारत सरकार
जल शक्ति मंत्रालय
केन्द्रीय भूमि जल बोर्ड
उत्तर पश्चिमी क्षेत्र, चण्डीगढ़

Government of India
Ministry of Jal Shakti
Central Ground Water Board
North Western Region, Chandigarh

File No. CGWB/NWR/Chem.Lab./2023- **Dated-11.08.2023**

TO WHOM IT MAY CONCERN

This is to certify that **Ms. Ekta Siwatch D/o Mr. Bhoop Singh Siwatch** has participated in the Skill Development Initiative (Training Program) of CGWB, NWR, Chandigarh from **26.06.2023 to 11.08.2023** under the supervision of **Sh. Kuldeep Gopal Bhartariya, Sc-B (Chemist).** She carried out analysis of groundwater samples following Standard Operating Procedure using conventional and modern instruments. Skills were developed in various hydro-chemical aspects of groundwater, including data generation in compliance with validation protocols and report writing.

Akhanna 11/8/23

(Anurag Khanna)
Regional Director

Address:

Ms. Ekta Siwatch
D/o Mr. Bhoop Singh Siwatch
Kurukshetra University,
Kurukshetra,
Haryana.

CHANDIGARH POLLUTION TESTING LABORATORY
(Environmental Monitoring, EIA, MSC & TSP)
NABET Accredited EIA Consultant

H.O. [illegible]
Lab [illegible]
E-mail [illegible]
Website www.cptl.co.in

CPTL/Trg/2023/08/02

Dated: 05-08-2023

CERTIFICATE OF TRAINING

This is to certify that Ms. Anshu Sharma D/o. Sh. Puran Chand as sponsored by Kurukshetra University, Department of Environmental Studies vide letter no. 2734 dated 07-06-2023 has satisfactorily undergone Summer Training from 20th June to 5th August 2023 in the Discipline of Environment Impact Assessment Studies related to development projects in the EIA-Division of this organization. She is relieved w.e.f. the afternoon of 5th August 2023.

We wish her success in her future endeavors.

Sd/-

Authorized Signatory

Chandigarh Pollution Control Committee

Paryavaran Bhawan, Madhaya Marg, Sector 19-B,Chandigarh- 160019

CPCC/Lab/21/Trng./2023/2359 Dated: 17-08-23

TO WHOMSOEVER IT MAY CONCERN

It is certified that **Ms. Alka Yadav,** student of M.Sc. Environment Sciences, Institute of Environmental Studies, **Kurukshetra University, Kurukshetra**, Haryana has successfully undergone training from **01.07.2023** to **15.08.2023** at Chandigarh Pollution Control Committee on **Water Quality Assessment of Sukhna Lake in Chandigarh.**

During the training period, she was punctual, hardworking and her conduct was good. She bears a good moral character.

I wish her all success in the future.

(Arulrajan P., IFS)
Member Secretary

Phone : 0172-2700149, EPABX : 0172-2700311, e -mail : cpcc-chd@nic.in

TRAINING CERTIFICATE

This is to certify that Ms. Raunak Soni D/o Sh. Bhoop Singh, bearing Registration No. 22-UD-552 & Roll No. 2022009822, a student of M.Sc. Environmental Science, Under Kurukshetra University, Kurukshetra worked under my supervision during her offline Summer internship Research Program period w.e.f. 20/06/2023 to 06/08/2023 under the office of Public Health Engineering Sub Division No.3, Fatehabad. During her internship she has demonstrated her skills with self-motivation to learn new skills.

We wish her all the best for her upcoming career.

Signature with seal of Issuing Authority

Satpal Rose
Sub Divn. Engineer
P.H. Engg. SDivn.No 3
FATEHABAD

Place:- Fatehabad.
Date:- 07/08/2023

TULIP

CERTIFICATE OF COMPLETION

This is to certify that Ms. SAHIBA a graduate of Kurukshetra University Kurukshetra has successfully completed her Internship with Municipal Corporation Rohtak from 27-06-2023 to 10-08-2023. During the period of Internship, she worked and assisted the Municipal Corporation Rohtak in research/design/fieldwork related to Waste Management, Bio Compost Training and Ward Wise IEC Activity with due diligence and commitment.

Date :23/08/23

Ajay Kumar
Municipal Commissioner

CERTIFICATE

This is to certify that the project entitled **'RAINFALL ESTIMATION USING DWR'** is a bonafide work done by Ms.NEETU , a summer trainee, M.Sc,3rd Semester, Kurukshetra University, Kurukshetra under my supervision at Meteorological Centre,Chandigarh.

04/08/2023

(A K SINGH)
(Scientist)
Meteorological Centre, Chandigarh.

(ए.के. सिंह)
(A.K. SINGH)
वैज्ञानिक/Scientist
मौसम केंद्र, Meteorological Centre,
सैक्टर-39-सी, चंडीगढ़-160036
Sector-39-C, Chandigarh-160036

HARYANA POWER GENERATION CORPORATION LIMITED
Regd. Office:--- C- 7, Urja Bhawan, Sector-6, Panchkula
Corporate Identity Number: U40207HR1997SGC033517

Ref. No. 02-PG

IN PLANT TRAINING CERTIFICATE

This is certified that **Mr. SACHIN** bearing Roll No. **2022009826** a student of M.Sc. Environmental Science in **KURUKSHETRA UNIVERSITY, KURUKSHETRA** has undergone vocational training at "Civil Maintenance Division" Panipat Thermal Power Station, Panipat from **June 26, 2023** to **August 14, 2023** during this period his performance is found very good.

Place:- PTPS, Panipat
Date:- August 21, 2023

Executive Engineer
Training Division
P.T.P.S. H.P.G.C.L. Panipat

CHANDIGARH POLLUTION TESTING LABORATORY
(Environmental Monitoring, EIA, NOC, ETP, STP)
NABET Accredited EIA Consultant

H.O #372, Sector 15-A, Chandigarh-160 015
Phone : 0172-4669256
Lab E-126, Phase-VII, Indl. Area, Mohali - 160 055
Phone : 0172-5080312
E-mail cptle126@gmail.com, lab@cptl.co.in
Website www.cptl.co.in

CPTL/Trg/2023/08/01

Dated: 05-08-2023

CERTIFICATE OF TRAINING

This is to certify that Ms. Ritika D/o. Sh. Raghubir as sponsored by Kurukshetra University, Department of Environmental Studies vide letter no. 2733 dated 07-06-2023 has satisfactorily undergone Summer Training from 20th June to 5th August 2023 in the Discipline of Environment Impact Assessment Studies related to development projects in the EIA-Division of this organization. She is relieved w.e.f. the afternoon of 5th August 2023.

We wish her success in her future endeavors.

CPTL Plot No. E-126, Indl. Area, Phase-7 MOHALI

Sd/-

Authorized Signatory

CERTIFICATE

This is to certify that the project entitled "Utilization of Satellites in Weather Services" is a bonafide record of the work done by ANKUR SHARMA, a summer trainee, M.SC(Environmental Science), final year, Kurukshetra university , Kurukshetra under my supervision at Meteorological Cetnre, Chandigarh.

04/08/2023

(A K SINGH)

(Scientist, Meteorological Cetnre, Chandigarh)

(ए.के. सिंह)
(A.K. SINGH)
वैज्ञानिक/Scientist
मौसम केंद्र, Meteorological Centre,
सैक्टर-39-सी, चं.गढ़-160036
Sector-39-C, Chandigarh-160036

My Certificate

CPCB

LiFE Lifestyle For Environment

G20

केन्द्रीय प्रदूषण नियंत्रण बोर्ड
CENTRAL POLLUTION CONTROL BOARD

No. C-11012/5/Misc/2011/Water Lab

Dated: 04.08.2023

TO WHOM IT MAY CONCERN

This is to certify that **Miss. Sonal D/o Sh. Surender Singh** student of M.Sc. Environment Science at Kurukshetra University, Kurushetra (Haryana) has undertaken Summer Training work on **"Assessment of physico-chemical study in soil and sludge sample"** at Soil & Solid Waste Laboratory a Sub Division of Central Water Laboratory of Central Pollution Control Board, Delhi from 20th June to 04th August 2023.

During the training she has learnt analysis of Soil & Sludge samples by different methods like gravimetric, titrimetric, colorimetric etc. She has also worked on the laboratory instruments like pH Meter, Conductivity meter, Flame photometer, UV-Visible Spectrophotometer.

She has completed the training satisfactorily. During this period she proved herself to be hard working, sincere and I found her very enthusiastic, attentive and regular in her training work.

I wish her all the success for her bright future.

4/8/23

(Dr. K. Ranganathan)
Scientist 'E'/Add. Director
Divisional Head
Water & Instrumentation Laboratory

डॉ. के. रंगनाथन / Dr. K. Ranganathan
वैज्ञानिक 'ई' / Scientist 'E'
प्रभारी जल एवं उपकरणशाला प्रयोगशाला
Dy. Head-Water & Instrumentation Laboratory
केन्द्रीय प्रदूषण नियंत्रण बोर्ड
Central Pollution Control Board

'परिवेश भवन' पूर्वी अर्जुन नगर, दिल्ली-110032
Parivesh Bhawan, East Arjun Nagar, Delhi-110032
दूरभाष/Tel : 43102030, 22305792, वेबसाईट/Website : www.cpcb.nic.in

CHANDIGARH POLLUTION TESTING LABORATORY
(Environmental Monitoring, EIA, NOC, ETP, STP)
NABET Accredited EIA Consultant

H.O. #372, Sector 15-A, Chandigarh-160 015
Phone : 0172-4666029
Lab E-126, Phase-VII, Ind. Area, Mohali - 160059
Phone : 0172-5080312
E-mail cptl126@gmail.com, info@cptl.co.in
Website www.cptl.co.in

CPTL/Trg/2023/08/03 Dated: 05-08-2023

CERTIFICATE OF TRAINING

This is to certify that Ms. Shalu D/o. Sh. Anil as sponsored by Kurukshetra University, Department of Environmental Studies vide letter no. 2735 dated 07-06-2023 has satisfactorily undergone Summer Training from 20th June to 5th August 2023 in the Discipline of Environment Impact Assessment Studies related to development projects in the EIA-Division of this organization. She is relieved w.e.f. the afternoon of 5th August 2023.

We wish her success in her future endeavors.

CPTL Plot No. E-126, Ind. Area, Phase-7 MOHALI

Sd/-

5/8/23

Authorized Signatory

To Whom So Ever It May Concern

This is to certify that **Miss. Simran**, a student from Kurukshetra University, Haryana, has successfully completed her Summer Training on **"Agriculture Pollution and its Management"** w.e.f. 03-07-2023 to 16-08-2023 under Department of agriculture and Farmer Welfare, Rewari.

We wish Miss. Simran all success in her carrier.

No.- 3074

Date: 29-08-2023

Deputy Director,
Agriculture and Farmer Welfare Department,
Rewari

CHANDIGARH POLLUTION TESTING LABORATORY
(Environmental Monitoring, EIA, NOC, ETP, STP)
NABET Accredited EIA Consultant

H.O.
Lab
E-mail
Website www.cptl.co.in

CPTL/Trg/2023/08/04

Dated: 05-08-2023

CERTIFICATE OF TRAINING

This is to certify that Ms. Purnima D/o. Sh. Yogendra as sponsored by Kurukshetra University, Department of Environmental Studies vide letter no. 2736 dated 07-06-2023 has satisfactorily undergone Summer Training from 20th June to 5th August 2023 in the Discipline of Environment Impact Assessment Studies related to development projects in the EIA-Division of this organization. She is relieved w.e.f. the afternoon of 5th August 2023.

We wish her success in her future endeavors.

Sd/-

CPTL MOHALI

Authorized Signatory

भारत सरकार
जल शक्ति मंत्रालय
केन्द्रीय भूमि जल बोर्ड
उत्तर पश्चिमी क्षेत्र, चण्डीगढ़

Government of India
Ministry of Jal Shakti
Central Ground Water Board
North Western Region, Chandigarh

File No. CGWB/NWR/Chem.Lab./2023- **Dated-11.08.2023**

TO WHOM IT MAY CONCERN

This is to certify that **Ms. Ritu Rani D/o Mr. Ami Lal** has participated in the Skill Development Initiative (Training Program) of CGWB, NWR, Chandigarh from **26.06.2023 to 11.08.2023** under the supervision of **Sh. Kuldeep Gopal Bhartariya, Sc-B (Chemist).** She carried out analysis of groundwater samples following Standard Operating Procedure using conventional and modern instruments. Skills were developed in various hydro-chemical aspects of groundwater, including data generation in compliance with validation protocols and report writing.

11/8/23

(Anurag Khanna)
Regional Director

Address:

Ms. Ritu Rani
D/o Mr. Ami Lal
Kurukshetra University,
Kurukshetra,
Haryana.

CERTIFICATE

This is to certify that the project entitled **"DWR & Upper Air Wind Observations"** is a bonafide record of the work done by Palak, a summer trainee ,M.Sc Environmental Science (2022-2024) , Kurukshetra University, Kurukshetra under my supervision at Meteorological Centre, Indian Meteorological Department ,Chandigarh.

04/08/2023

(A K Singh)

(Scientist)

Meteorological Centre, Chandigarh

(ए.के. सिंह)
(A.K. SINGH)
वैज्ञानिक/Scientist
मौसम केंद्र, Meteorological Centre,
सैक्टर-39-सी, चंडीगढ़-160036
Sector-39-C, Chandigarh-160036

भारत सरकार
GOVERNMENT OF INDIA
भारत मौसम विज्ञान विभाग
INDIA METEOROLOGICAL DEPARTMENT

ई-मेल : ts.chandimet@gmail.com
दूरभाष : 0172 - 2629984, 2920224
फैक्स : 0172 - 2629984, 2629981

संख्या सी.एच.टी 50/ 4002
दिनांक: 04 अगस्त ,2023,

निदेशक का कार्यालय,
Office of the Director
मौसम केन्द्र,
Meteorological Centre
सैक्टर 39 - सी, चंडीगढ़ - 160036
Sector 39-C,Chandigarh - 160036

TO WHOMSOEVER IT MAY CONCERN

This is to certify that **Ms. Prachi** student of M.Sc (Environmental Studies) from the Institute of Kurukshetra University, Kurukshetra - Haryana, has successfully completed her internship from June 19, 2023 to August 04, 2023 with Meteorological Centre Chandigarh, India Meteorological Department, Ministry of Earth Science, Government of India.

In her capacity as an intern she has successfully completed her summer project on **"Iot Based Automatic Weather Services"** under the guidance of **Sh. Ajay Kumar Singh, Scientist 'E', and Meteorological Center Chandigarh**. Her performance as an intern was excellent.

I wish her all the best for her future endeavors.

04/8/2023

(मनमोहन सिंह) / (Manmohan Singh)
वैज्ञानिक 'एफ' व प्रमुख / Scientist 'F' & Head
मौसम केन्द्र चंडीगढ / Meteorological Centre Chandigarh

(MANMOHAN SINGH)
प्रमुख / निदेशक
Head/Director
मौसम केन्द्र, चण्डीगढ़
Meteorological Centre, Chandigarh

CERTIFICATE

This is to certify that the project entitled "**Doppler Weather Radar & Weather Services**" is a Bonafide record of the work done by Sourabh, a summer trainee, MSc (Env. Science) final year, Kurukshetra university, Kurukshetra under my supervision at India Meteorological Department, Chandigarh.

04/08/2023

(A K Singh)

Scientist, Meteorological Centre

IMD, Chandigarh

(ए.के. सिंह)
(A.K. SINGH)
वैज्ञानिक/Scientist
मौसम केंद्र, Meteorological Centre,
सैक्टर-39-सी, चंडीगढ़-160036
Sector-39-C, Chandigarh-160036

CERTIFICATE OF INTERNSHIP

सं. संगठन प्रभाग/2023
भारत सरकार
भारत मौसम विज्ञान विभाग
का कार्यालय
मौसम विज्ञान महानिदेशक
मौसम भवन, लोदी रोड
नई दिल्ली - 110 003 (भारत)

No. Organization Div/2023
Government of India
India Meteorological Department
Office of the
Director General of Meteorology
Mausam Bhawan, Lodi Road
New Delhi - 110 003 (India)
Dated: 18.08.2023

TO WHOMSOEVER IT MAY CONCERN

This is to certify that **Miss Shivani** from Kurukshertra University, kurukshetra has successfully completed her internship in Satellite Division of India Meteorological Department, Lodi Road, New Delhi from 4th July to 18th August 2023.

She has worked on project titled **"EFFECT OF MONSOON ON AIR QUALITY OF A METROPOLITAN CITY"**. As part of the project, she has shown great efforts and dedication in carrying out all assigned duties, which included gathering and interpreting data, analyzing and reporting results to concerned departments, to name a few.

During her internship, she has demonstrated her skills with self-motivation to learn new skills. Her performance exceeded our expectations and she was able to complete the work on time.

We wish her all the best for her upcoming career.

R.K. Giri

(Dr. R.K. Giri)
Scientist-F
Rk.giri7@imd.gov.in
Phone: 91-11-24643025

CERTIFICATE

WATER TECH ENGINEERS

New Delhi, India

Dt.17/08/2023

TO WHOME SOEVER IT MAY CONCERN

This is to certified that Ms. Sakshi student of M.Sc. (Environment Science) from the Institute of Environment Studies, Kurukshetra University, Kurukshetra Haryana, has successfully completed her internship from 1st July 2023 to 15th August 2023 at Water Tech Engineers, An Environment Consulting, Engineering & Projects Manufacturers) New Delhi, India. From 1st July 2023 to 15th August. She has worked on a project title - Wastewater Treatment -STP Plant-MBBR-Technology, How the industrial wastewater management control in - an Industrial unit, and she is very self-motivated attitude to learn new things.

I wish her all the best for her future endeavors

For Water Tech Engineers

Anil Kr. Sisodia
Chief Operations

For Water Tech Engineers

Proprietor

Company Stamp

CERTIFICATE

This is to certify that the project entitled "**MSLP & its relationship with DWR Reflectivity**" is a bonafide record of the work done by Sakshi, a summer trainee, M.Sc Environmental Science (2022-2024), Kurukshetra University, Kurukshetra under my supervision at Meteorological Centre, Indian Meteorological Department, Chandigarh.

04/08/2023

(AK Singh)

(Scientist)

Meteorological Centre, Chandigarh

(ए.के. सिंह)
(A.K. SINGH)
वैज्ञानिक/Scientist
मौसम केंद्र, Meteorological Centre,
सैक्टर–39–सी, चंडीगढ़–160036
Sector-39-C, Chandigarh-160036

केन्द्रीय प्रदूषण नियंत्रण बोर्ड
CENTRAL POLLUTION CONTROL BOARD
पर्यावरण, वन एवं जलवायु परिवर्तन मंत्रालय भारत सरकार
MINISTRY OF ENVIRONMENT FOREST & CLIMATE CHANGE GOVT OF INDIA

No. C-11012/5/Misc/2011/Water Lab

Dated: 04.08.2023

TO WHOM IT MAY CONCERN

This is to certify that **Miss. Salony D/o Sh. Om Parkash** student of M.Sc. Environment Science at Kurukshetra University, Kurushetra (Haryana) has undertaken Summer Training work on **"Analytical Methods for Waste Water analysis"** at Water Laboratory (Waste Water Laboratory) of Central Pollution Control Board, Delhi from 20th June to 04th August 2023.

During the training she has learnt analysis of waste water samples by different Physico-chemical methods such as Gravimetric, Titrimetric, Colorimetric, Distillation etc. She has also worked on the laboratory instruments like pH Meter, Conductivity meter, Flame photometer, UV-Visible Spectrophotometer.

She has completed the training satisfactorily. During this period she proved herself to be hard working, sincere and I found her very enthusiastic, attentive and regular in her training work.

I wish her all the success for her bright future.

4/8/23

(Dr. K. Ranganathan)
Add. Director & Divisional Head
Water & Instrumentation Laboratory

डॉ. के. रंगनाथन / Dr. K. Ranganathan
वैज्ञानिक 'एफ' / Scientist 'F'
प्रभारी जल एवं उपकरणीकरण प्रयोगशाला
Dy. Head-Water & Instrumentation Laboratory
केन्द्रीय प्रदूषण नियंत्रण बोर्ड
Central Pollution Control Board
पर्यावरण, वन एवं जलवायु परिवर्तन मंत्रालय, भारत सरकार
Min. Environment, Forest & Climate Change, Govt. of India
परिवेश भवन, पूर्वी अर्जुन नगर, दिल्ली-110032
Parivesh Bhawan, East Arjun Nagar, Delhi-110032

CERTIFICATE

WATER TECH ENGINEERS
New Delhi, India

Dt.17/08/2023

TO WHOME SOEVER IT MAY CONCERN

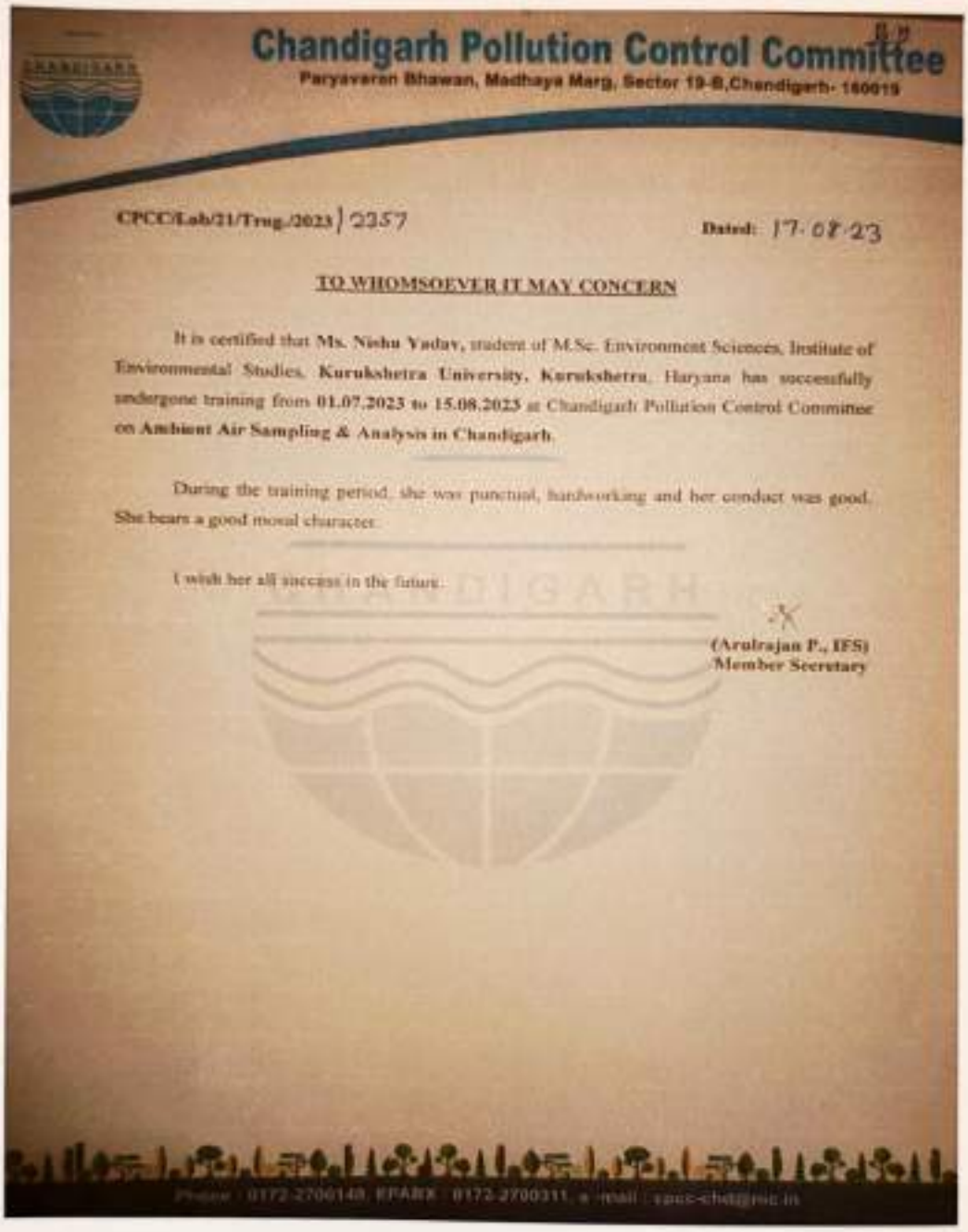
This is to certified that Ms. Chanchal student of M.Sc. (Environment Science) from the Institute of Environment Studies, Kurukshetra University, Kurukshetra Haryana, has successfully completed her internship from 1st July 2023 to 15th August 2023 at Water Tech Engineers, An Environment Consulting, Engineering & Projects Manufacturers, New Delhi, India. From 1st July 2023 to 15th August. She has worked on a project title – Industrial Waste Management. How the industrial waste management control in – an Industrial unit, and she is very self-motivated attitude to learn new things.
I wish her all the best for her future endeavors

For Water Tech Engineers

Anil Kr. Sisodia
Chief Operations

Company Stamp

CERTIFICATE



Chandigarh Pollution Control Committee

Paryavaran Bhawan, Madhaya Marg, Sector 19-B,Chandigarh- 160019

CPCC/Lab/21/Trng./2023/2357 Dated: 17.08.23

TO WHOMSOEVER IT MAY CONCERN

It is certified that **Ms. Nishu Yadav**, student of M.Sc. Environment Sciences, Institute of Environmental Studies, **Kurukshetra University, Kurukshetra**, Haryana has successfully undergone training from **01.07.2023 to 15.08.2023** at Chandigarh Pollution Control Committee on **Ambient Air Sampling & Analysis in Chandigarh**.

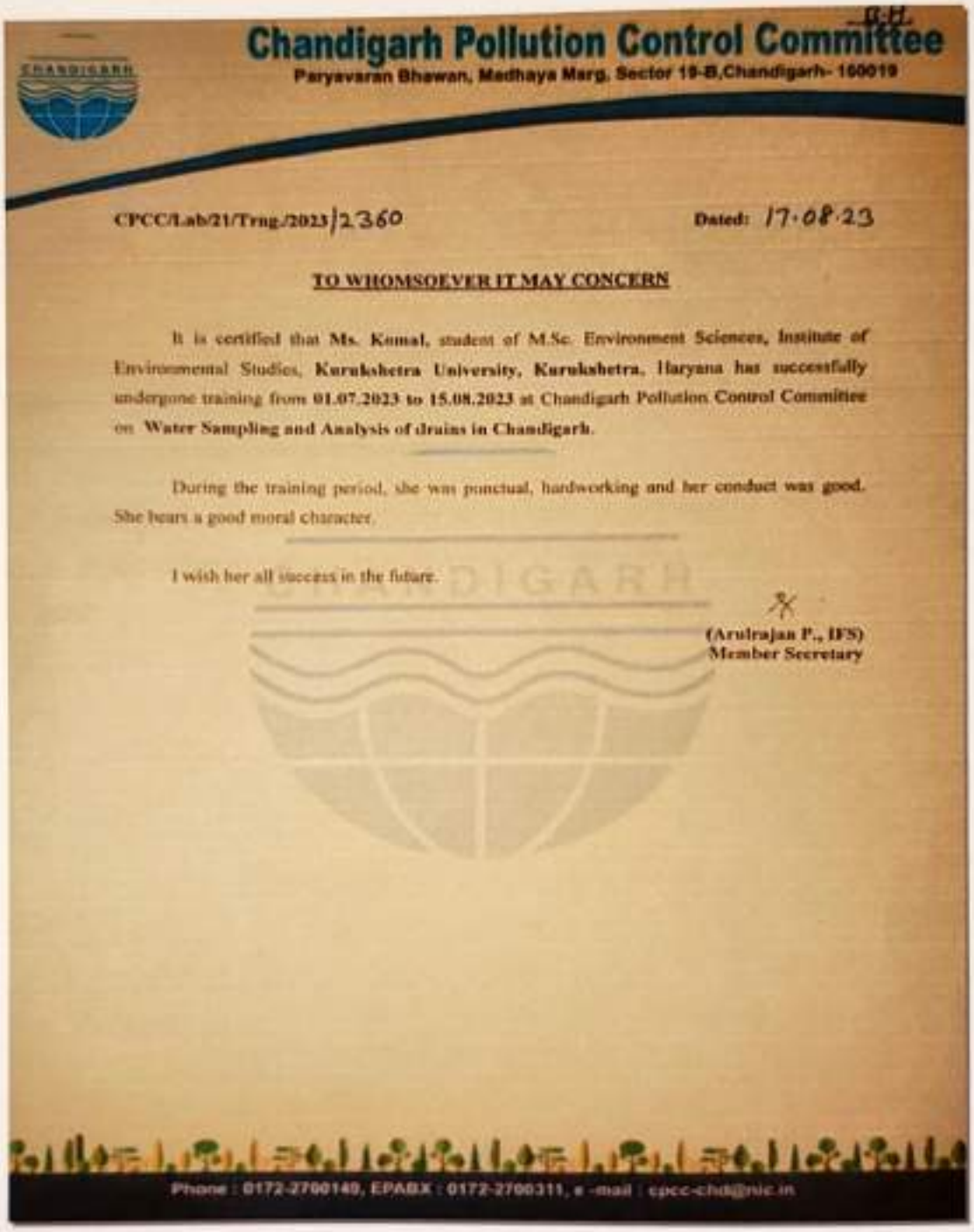
During the training period, she was punctual, hardworking and her conduct was good. She bears a good moral character.

I wish her all success in the future.

(Arulrajan P., IFS)
Member Secretary

Phone : 0172-2700149, FPABX : 0172-2700311, e-mail : cpcc-chd@nic.in

CERTIFICATE



Chandigarh Pollution Control Committee

Paryavaran Bhawan, Madhaya Marg, Sector 19-B,Chandigarh- 160019

CPCC/Lab/21/Trng/2023/2360 Dated: 17.08.23

TO WHOMSOEVER IT MAY CONCERN

It is certified that **Ms. Komal**, student of M.Sc. Environment Sciences, Institute of Environmental Studies, **Kurukshetra University, Kurukshetra, Haryana has successfully** undergone training from **01.07.2023 to 15.08.2023** at Chandigarh Pollution Control Committee on **Water Sampling and Analysis of drains in Chandigarh.**

During the training period, she was punctual, hardworking and her conduct was good. She bears a good moral character.

I wish her all success in the future.

(Arulrajan P., IFS)
Member Secretary

Phone : 0172-2700140, EPABX : 0172-2700311, e -mail : cpcc-chd@nic.in

ENVIROCHEM TESTING LAB
& Research Centre

(An ISO 9001 : 2015, ISO 14001 : 2015, ISO 18001 : 2007 Certified Lab)
Plot No. 165, 1st Floor, Sector-25, Part-II, HUDA, Panipat-132103, Hr.
M.: +91 90348 91129, 89501 75388, 96719 56782
Email : envirochemtestinglab@gmail.com Web. : www.etlrc.com

Dated: 09.08.2023

TO WHOM IT SO EVER IT MAY CONCERN

It is certified that Miss Vimal D/o Sh. Randhir Singh has attended internship from 28th June 2023 to 9th August 2023 as a trainee for testing in the Field of Drinking water, wastewater, and Sophisticated Instrumentation handling at ENVIROCHEM TESTING LAB & RESEARCH CENTRE.

We wish for success her future endeavour.

Envirochem Testing Lab & Research Centre Authorised Sign 9/8/2023 PANIPAT

Mrs. Poonam Kalsyan (Technical Manager)
Envirochem Testing Lab & Research Centre

ए पी सी पी एल
APCPL

अरावली पावर कम्पनी प्राइवेट लिमिटेड
(एनटीपीसी, एचपीजीसीएल एवं आईपीजीसीएल का संयुक्त उद्यम)
Aravali Power Company Private Limited
(A joint venture of NTPC, HPGCL AND IPGCL)
CIN No.:U40105DL2006PTC156884

Ref No. APCPL/HR/EDC/2023-24

Date: 17/08/2023

TO WHOM SO EVER IT MAY CONCERN

This is to certify that **Miss. Anjali**, a student from **Kurukshetra University, Haryana** successfully completed her Summer Internship Training on **EMG** Development of APCPL, Jharli, Jhajjar, w.e.f. **03/07/2023 to 16/08/2023** under the guidance of **Sh. S.K.Agarwal, AGM (EMG&AUD).**

We wish **Miss. Anjali** all success in her career.

17/8/2023

(Rajesh Kumar Nagrath)
Manager (HR)

Miss. Anjali
Kurukshetra University, Haryana

एनटीपीसी NTPC HPGCL

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WATER TECH ENGINEERS

New Delhi, India

Dt.17/08/2023

TO WHOME SOEVER IT MAY CONCERN

This is to certified that Ms. Manish student of M.Sc. (Environment Science) from the Institute of Environment Studies, Kurukshetra University, Kurukshetra Haryana, has successfully completed her internship from 1st July 2023 to 15th August 2023 at Water Tech Engineers, An Environment Consulting, Engineering & Projects Manufacturers) New Delhi, India. From 1st July 2023 to 15th August. She has worked on a project title - Industrial Waste Management. How the industrial waste management control in - an Industrial unit, and she is very self-motivated attitude to learn new things.

I wish her all the best for her future endeavors

For Water Tech Engineers

Atul Kr. Sisodia
Chief Operations

Company Stamp

B.H

Chandigarh Pollution Control Committee

Paryavaran Bhawan, Madhaya Marg, Sector 19-B,Chandigarh- 160019

CPCC/Lab/21/Trng./2023/2358 Dated: 17.08.23

TO WHOMSOEVER IT MAY CONCERN

It is certified that **Ms. Ankita,** student of M.Sc. Environment Sciences, Institute of Environmental Studies, **Kurukshetra University, Kurukshetra,** Haryana has successfully undergone training from **01.07.2023 to 15.08.2023** at Chandigarh Pollution Control Committee on **Water sampling and analysis of STP's in Chandigarh.**

During the training period, she was punctual, hardworking and her conduct was good. She bears a good moral character.

I wish her all success in the future.

(Arulrajan P., IFS)
Member Secretary

Phone : 0172-2700149, EPABX : 0172-2700311, e -mail : cpcc-chd@nic.in

CERTIFICATE OF INTERNSHIP

सं. संगठन प्रभाग/2023
भारत सरकार
भारत मौसम विज्ञान विभाग
का कार्यालय
मौसम विज्ञान महानिदेशक
मौसम भवन, लोदी रोड
नई दिल्ली - 110 003 (भारत)

No. Organization Div/2023
Government of India
India Meteorological Department
Office of the
Director General of Meteorology
Mausam Bhawan, Lodi Road
New Delhi - 110 003 (India)
Dated: 18.08.2023

TO WHOMSOEVER IT MAY CONCERN

This is to certify that **Miss Kajal** from Kurukshertra University, kurukshetra has successfully completed her internship in Satellite Division of India Meteorological Department, Lodi Road, New Delhi from 4th July to 18th August 2023.

She has worked on project titled **"WEATHER CONDITIONS DIFFRENCES BETWEEN A COASTAL AND A LANDLOCKED REGION"**. As part of the project, she has shown great efforts and dedication in carrying out all assigned duties, which included gathering and interpreting data, analyzing and reporting results to concerned departments, to name a few.

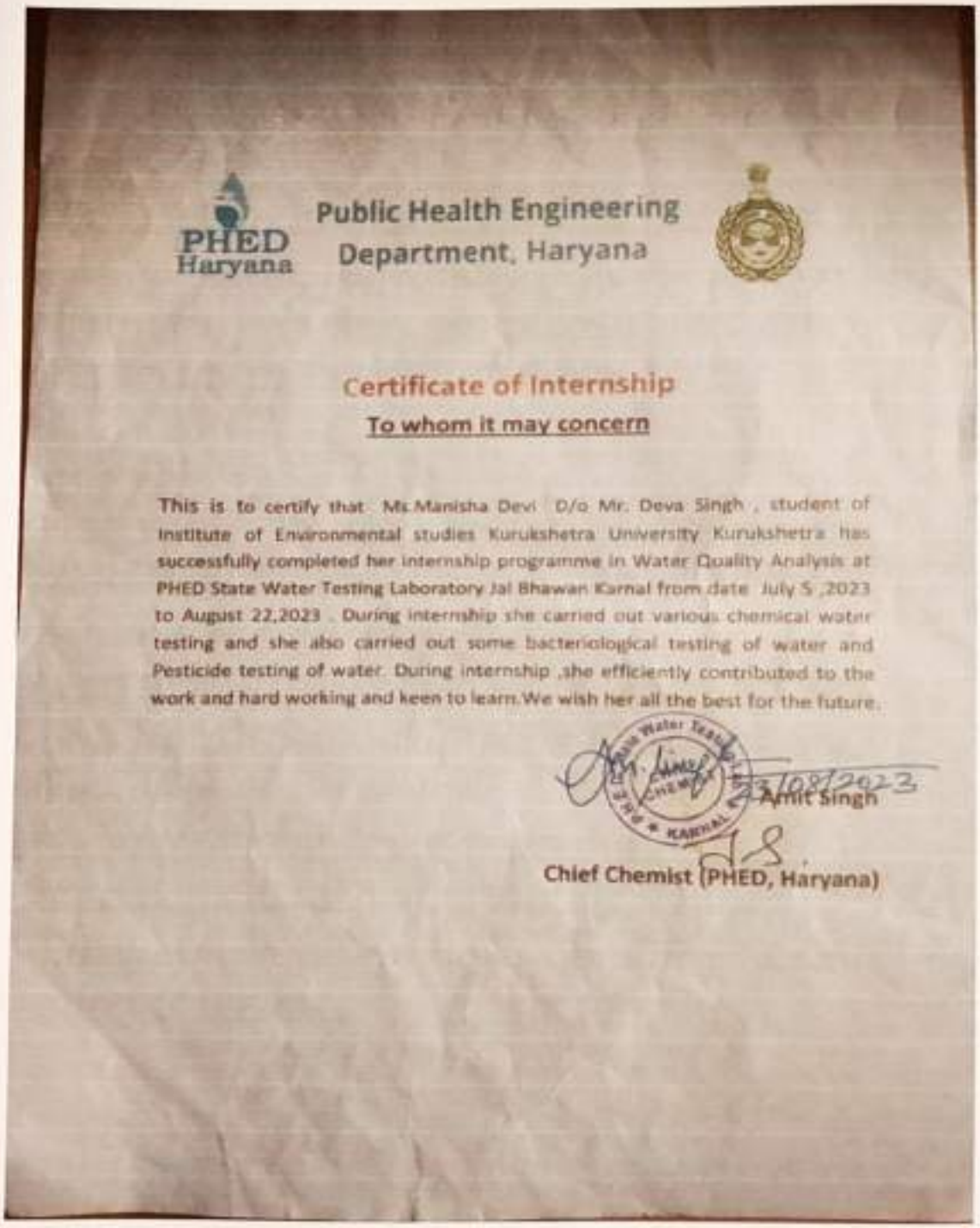
During her internship, she has demonstrated her skills with self-motivation to learn new skills. Her performance exceeded our expectations and she was able to complete the work on time.

We wish her all the best for her upcoming career.

R.K. Giri

(Dr. R.K. Giri)
Scientist-F
Rk.giri71@imd.gov.in
Phone: 011-24632075

Training Certificate:-



PHED Haryana

Public Health Engineering Department, Haryana

Certificate of Internship

To whom it may concern

This is to certify that Ms.Manisha Devi D/o Mr. Deva Singh , student of Institute of Environmental studies Kurukshetra University Kurukshetra has successfully completed her internship programme in Water Quality Analysis at PHED State Water Testing Laboratory Jal Bhawan Karnal from date July 5 ,2023 to August 22,2023 . During internship she carried out various chemical water testing and she also carried out some bacteriological testing of water and Pesticide testing of water. During internship ,she efficiently contributed to the work and hard working and keen to learn.We wish her all the best for the future.

Dated: 23/08/2023

Amit Singh

Chief Chemist (PHED, Haryana)

Perfact

PERFACT GROUP

4th September, 2023

TO WHOMSOEVER IT MAY CONCERN

This is to certify that **Mr. Abhishek Singh,** a student of **Institute of Environmental Studies, Kurukshetra University, Kurukshetra; Msc- EVS , Roll Number 2022009801(19) ,** has successfully completed his minimum 45-day mandatory internship with **Perfact Group, Janakpuri office** from **29th June, 2023- 18th August, 2023.**

His department of working was **Environmental Impact Assessment (E.I.A.)**

The title of his project was **"Assessment of Dust Pollution from Construction Project in Delhi and NCR Region."**

During the entire period of his internship, we found him to be a focused, honest and a dedicated professional who handled the work assigned to him with zeal.

We wish him all the very best in all his future endeavors.

Regards,

5th Floor, NN Mall, Sector 3, Rohini, New Delhi - 110085 | +91 11 49281360 | www.perfactgroup.com | info@perfactgroup.com

CCS Haryana Agricultural University
College of Agriculture, Bawal, Rewari,
Haryana 123501
Email: principalbawal@gmail.com,
Phone: 01284-297124 (O), 9416498700 (M)

No.: COAB/23/2943 Dated: 5-08-2023

Subject: Summer Training (Offline Summer Internship Research Programme).

Sir,

It is certified that Ms. Alka, student of M.Sc. (Environmental Science), Institute of Environmental Science, Kurukshetra University, Kurukshetra has completed her 45 days summer training from this College in the laboratory of Soil Science on the topic "Soil Analysis from Organic, Inorganic and Conventional Farming System of South-West Haryana".

During summer training, Ms. Alka worked sincerely and did all analysis in the laboratory. She has also submitted a report of the training programme. A copy of the same will be sent to the Institute of Environmental Science, Kurukshetra University, Kurukshetra.

5.8.23

(Dr. Naresh Kaushik)
Principal
CCS Haryana Agricultural University
College of Agriculture, Bawal (123501)
Rewari (Haryana) India

The Director
Institute of Environmental Science
Kurukshetra University, Kurukshetra

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IN PLANT TRAINING CERTIFICATE

This is certified that **Mr. ANAND KUMAR** bearing Roll No. **2022009804** a student of M.Sc. Environmental Science in **KURUKSHETRA UNIVERSITY, KURUKSHETRA** has undergone vocational training at "Civil Maintenance Division" Panipat Thermal Power Station, Panipat from **June 26, 2023** to **August 14, 2023** during this period his performance is found very good.

Place:- PTPS, Panipat
Date:- August 21, 2023

Executive Engineer
Training Division
P.T.P.S. H.P.G.C.L. Panipat

WATER TECH ENGINEERS

New Delhi, India

Dt.17/08/2023

TO WHOME SOEVER IT MAY CONCERN

This is to certified that Ms. Pooja student of M.Sc. (Environment Science) from the Institute of Environment Studies, Kurukshetra University, Kurukshetra Haryana, has successfully completed her internship from 1st July 2023 to 15th August 2023 at Water Tech Engineers, An Environment Consulting, Engineering & Projects Manufacturers) New Delhi, India. From 1st July 2023 to 15th August. She has worked on a project tittle - Wastewater Treatment -STP Plant-MBR Technology How the industrial wastewater management control in - an Industrial unit, and she is very self-motivated attitude to learn new things.

I wish her all the best for her future endeavors

For Water Tech Engineers

Atul Kr. Sisodia
Chief Operations

Proprietor

Company Stamp

THE HISAR-JIND CO-OPERATIVE MILK PRODUCERS UNION LTD
MILK PLANT, JIND

Ref. No.MUHJ/Admn/2023/7723

Dated 23-8-23

TO WHOM IT MAY CONCERN

This is to certify that Ms. Serena, a student of M.Sc. Environmental Science 2nd Semester from Kurukshetra University, Kurukshetra has undergone practical training w.e.f 30.06.2023 to 10.08.2023 in Quality Control Section at Milk Plant Jind. During this period her work and conduct has been found Satisfactory.

She has completed her training successfully.

Chief Executive Officer
Milk Union Hisar-Jind.

(AN ISO 9001: 2015 & ISO 22000:2018 CERTIFIED UNIT)
PHONE: 01681-225776, 225772, 9896476401 TELE-FAX = 226266
E-mail: vitajind@gmail.com

CERTFICATE

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Ganpati Dairy Products Pvt. Ltd.

Sirsi Road, V.P.O.- Behal, District - Bhiwani (HR) Pin Code : 127028
Email : ganpatidairy@gmail.com

Ref. No. : GDPPL/TP/23-24/013 Dated : 03/08/2023

Training Certificate

TO WHOM IT MAY CONCERN

This is certified that **Reena Kumari D/O Mahipal**, A Student of **M.Sc. Environment Science** From Kurukshetra University, Kurukshetra (Haryana) has completed her training on **waste management of dairy** under the guidance of Mr. Paramjeet Singh at Ganpati Dairy Products Private Limited Behal From 19/06/2023 To 04/08/2023.

We wish all success in her life and future endeavor.

Director

Ganpati Dairy product Pvt Ltd

Environmental Science and Pollution Research (2023) 30:113335–113361
https://doi.org/10.1007/s11356-023-30357-9

RESEARCH ARTICLE

Groundwater hydrogeochemistry and non-carcinogenic health risk assessment in major river basins of Punjab, India

Lakhwinder Kaur[1,2,A] · Madhuri S. Rishi[2] · Bhagwan Singh Chaudhary[1] · Sakshi Sharma[2,3] · Sanjay Pandey[3]

Received: 2 February 2023 / Accepted: 25 September 2023 / Published online: 1 / October 2023

Abstract

The Indian Punjab state is drained by the four rivers, along with a well-connected network of canals, and is now dealing with a slew of water quality issues and problems. In this study, basin-wise hydrogeochemical modelling of 323 groundwater samples and identification of NO_3^- and F^- enrichment pathways in aquifer systems of Punjab were studied using different plots and multivariate statistics. To evaluate the groundwater quality and human health risks, an entropy-based water quality index and Monte Carlo simulation were used, respectively. Spatial distribution of NO_3^- indicated that its very high values were prominent in parts of southwestern Punjab falling under LSRB, along with few pockets in eastern and northeastern Punjab falling under MSRB and GRB. High NO_3^- values (> 45.0 mg/L) were found in 15.0% of Ravi River Basin (RRB) groundwater samples, 22.86% of Beas River Basin (BRB), 23.52% of Middle Sutlej River Basin (MSRB), 36.9% of Lower Sutlej River Basin (LSRB), and 21.31% of Ghaggar River Basin (GRB). The spatial distribution of NO_3^- revealed elevated concentrations (> 100 mg/L) in the southwestern part of Punjab, particularly in LSRB and localized pockets in the eastern and northeastern areas of Punjab within MSRB and GRB. High F^- concentration (> 1.5 mg/L) was observed in 15.12% and 21.31% groundwater samples of LSRB and GRB, respectively. Spatially southern parts falling under LSRB and GRB reflected high F^- content (> 1.5 mg/L) in groundwater. In LSRB, evaporative and anthropogenic processes influence the groundwater quality. The results of interionic relationships and statistical analysis revealed that NO_3^- has anthropogenic origin and that is being aggravated by leaching, the evaporation processes, animal excreta, septic tanks and irrigation return flows in LSRB and GRB, while F^- is geogenic in nature. Hazard index (HI) values in 14.63%, 22.2%, 24.6%, 49.58%, and 34.42% samples for adults and 21.95%, 27.7%, 42.0%, 72.3%, and 52.46% samples for children were higher than unity in RRB, BRB, MSRB, LSRB, and GRB, respectively. The basin-wise demarcation of various groundwater quality parameter and assessment of human health risk would be of significance for the management of water resources.

Keywords Agricultural activities · Groundwater quality · Nitrate and fluoride enrichment · Human health risk assessment · Monte Carlo simulation

Responsible Editor: Xianliang Yi

✉ Lakhwinder Kaur
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5 Centre for International Projects Trust, 95-C, BRS Nagar, Ludhiana 41013, India

Introduction

Water is a precious natural resource due to its multiple roles in propelling environmental, economic, and social cycles. The population of the world has multiplied over the past century, and lifestyle dynamics, together with urbanization and industrialization, have put stress on the limited freshwater resources (Velmurugan et al. 2020). The "UN World Water Development Report" (2022) stated that groundwater globally supplies 50% of domestic water demand including a vast majority of rural population that primarily relies on groundwater for drinking purposes. The majority of developing countries, including India, rely heavily on groundwater as a source of drinking water. Nearly 90% of rural population in

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https://doi.org/10.1007/s12040-023-02206-7

Geophysical signatures of the Saraswati River palaeochannel in a part of Kurukshetra district, Haryana, India

SUSHIL KUMAR[1,*], ASHISH JANGRA[1], NEPAL CHANDRA MONDAL[2,3], BHAGWAN SINGH CHAUDHARY[1], KRISHAN KUMAR[1], AAKASH DEEP[1], KAMAL[1] and SAVITA SINGH[1]

[1]*Department of Geophysics, Kurukshetra University Kurukshetra, Haryana 136 119, India.*
[2]*Electrical and Hollborne Geophysics Group, CSIR-National Geophysical Research Institute, Hyderabad 500 007, India.*
[3]*Academy of Scientific and Innovative Research (AcSIR), Ghaziabad, India.*
[*]*Corresponding author. e-mail: sushil_gpf@kuk.ac.in*

MS received 3 April 2023; revised 4 August 2023; accepted 21 September 2023

The palaeochannel is one of the promising features to hold a considerable amount of groundwater and acts as an underground reservoir for supplementing groundwater resources. There is a need for site-specific studies for exploring new areas for further groundwater prospecting in the wake of dwindling groundwater resources. The groundwater exploration studies have been conducted along and across the possible Saraswati River palaeochannel in a part of the Kurukshetra district of Haryana to understand the subsurface groundwater regime. Electrical resistivity tomography (ERT) surveys were conducted at Garhi Roran and Indhari villages of Kurukshetra district, Haryana. The ERT results indicate broadly three distinct lithological units up to the explored depth of 20 meters (m). The third layer showed relatively higher resistivity in comparison to the second layer. Therefore, vertical electrical sounding (VES) surveys were conducted in the study area at eight villages in the Kurukshetra district, including Garhi Roran and Indhari villages. The inverted model was correlated with the available lithological information for various lithological units. The maximum depth of investigation derived from the VES surveys was found to be 120 m. Further, a palaeo-path of high resistivity was delineated from 15 to 50 m depth and the width of the palaeochannel was interpreted as about 10–12 km. The hydrological data analysis of nearby bore wells shows a highly productive zone of good-quality groundwater. The analysis of Dar–Zarrouk (D–Z) parameters also indicates the presence of palaeochannel in the study area.

Keywords. Saraswati River palaeochannel; electrical resistivity survey; Dar–Zarrouk parameters; Kurukshetra.

1. Introduction

Palaeochannels are remnants of rivers or stream channels that flowed in the past and have been entirely filled or buried by younger fluvatile sediments (Wray 2009). Palaeochannel sediments, often comprised primarily of sand and gravel deposits with reduced clay and silt, can function as subsurface conduits due to their high porosity and

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Environ Monit Assess (2024) 196:548
https://doi.org/10.1007/s10661-024-12672-5

RESEARCH

Geophysical characterization of Saraswati River palaeochannel in parts of Yamuna Nagar and Kurukshetra districts of Haryana, India

Savita · B. S. Chaudhary · Sushil Kumar · S. Bhatnagar · Priyanka · Ayush Kesharwani · Anurag Khanna

Received: 28 December 2023 / Accepted: 25 April 2024 / Published online: 14 May 2024

Abstract Palaeochannels are remnants of rivers or stream channels filled with younger sediments over the period of time. In ancient times, these rivers/channels were thriving in phenomenal conditions, but due to frequent tectonic activities, they lost the direction of their original path and were gradually either lost or buried under thick beds of younger alluvium. Palaeochannels act as reservoirs for fresh groundwater since they are made up of coarser sediments and were formerly flowing rivers. Depending on the groundwater regime and local topography, these could either be saturated or dry. The palaeochannels have high groundwater potential if saturated. These are ideal sites for artificial groundwater recharge, if dry. The identification of palaeochannels becomes quite challenging if they are buried under thick deposits of finer younger sediments. In the present study, an attempt has been made to characterize the Saraswati River Palaeochannel in parts of Yamuna Nagar and Kurukshetra districts of Haryana by using surface and subsurface geophysical methods. Till date, the palaeochannels in this area were mainly discerned on the basis of remote sensing only; therefore, geophysical characterization of these palaeochannels has been attempted in this study. In surface geophysical methods, electrical resistivity surveys, especially gradient resistivity profiling (GRP) and vertical electrical sounding (VES), were conducted in the study area, while electrical and natural gamma logging was used as subsurface geophysical approaches to identify the coarser sands of buried palaeochannels. The main objective of the study was to characterize the Saraswati River palaeochannel and analyze the quality of the groundwater stored in the palaeochannel in the study area. The findings were compared with the well-log data and were found in good agreement.

Keywords Gradient resistivity profiling · Vertical electrical sounding · Schlumberger configuration · Electrical and natural gamma logging · Remote sensing

Savita · B. S. Chaudhary · S. Kumar
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Journal of the Indian Society of Remote Sensing (May 2024) 52(5):1045–1059
https://doi.org/10.1007/s12524-024-01858-8

RESEARCH ARTICLE

Geotechnical Characterization and PS-InSAR for Risk Analysis of Solang Landslide in Beas Valley, NW Himalaya: A Wake-Up Call!

Ramandeep Kaur[1,2] · Vikram Gupta[1,3] · Kapil Malik[4] · Bhagwan Singh Chaudhary[2]

Received: 20 June 2023 / Accepted: 20 March 2024 / Published online: 7 April 2024

Abstract
Landslide is one of the most common occurring natural disasters in the Himalayan terrain due to its rugged topography, steep slopes, and structural instability. The repercussions of landslides in Himalaya are often devastating, leading to loss of life, property, and infrastructure. Therefore, it is important to monitor landslides and reduce its consequences for which the state-of-the-art Persistent scatterers-Interferometric Synthetic Aperture Radar (PS-InSAR) technique is readily used nowadays. The present study illustrates a combined approach using PS-InSAR and a semi-quantitative empirical model for landslide risk micro-zonation utilizing the case study of Solang village (Himachal Pradesh, India). The analysis exhibits that a large part of the village is undergoing deformation with a subsidence rate of upto 80 mm/year near the crown portion of the landslide. The risk analysis indicates that ~ 50% of the buildings housing more than 100 people are under high to very high risk. To better understand the landslide phenomenon in the area, the study also investigates the detailed geomorphological and geological characterization of the area, geotechnical characters of the soil and rainfall pattern in the area. The present study highlights the scope of advanced geoinformatics techniques like InSAR in site-specific risk analysis of landslides and the need for mitigation in the Solang landslide zone.

Keywords Himalaya · Solang · Landslide · Risk · PS-InSAR

Introduction

Landslide is one of the natural mountain denudation processes in the Himalaya (Dortch et al., 2009; Korup et al., 2007). Generally, the down slope movement of slope material causes a change in slope geometry, relief and topography (Gilchrist et al., 1994; Li et al., 2020; Zeitler et al., 2001). However in the Himalayan terrain, the risk posed due to landslide and related mass movement has multiplied as the population has grown manifold and urbanization has encroached into unstable hillslopes (Petley, 2012). Consequently > 200 casualties and economic loss of the order of ~ 1 Billion US dollar are reported each year in the Himalaya (Naithani, 1999). There are numerous instances of landslides in the Himalaya which have destroyed entire villages, e.g. a large landslide triggered by 6.9 Mw Sikkim earthquake of 18 September 2011 destroyed one village in upstream of the Teesta Chu river whereby 120 villagers were reported missing (Martha et al., 2015); June 2013 Kedarnath disaster completely washed away Rambara village whereby > 1000 people were missing (Bhambri et al., 2016), 23 October 2013 Soldha landslide in Kangra district (Himachal Pradesh) destroyed 12 buildings along with ~ 29 ha of agricultural land (Mahajan et al., 2022), 2 August 2014 Jure landslide in Nepal wiped out the entire Jure village killing > 156 people (Lamichhane et al., 2021) and 6 September 2014 rock-fall destroyed the entire Sadal village (Jammu & Kashmir, India) comprising 75 houses and killing > 40 people (Kumar et al., 2017). However, in recent years, there is an increase in a number of these kind of landslide events in the Himalaya (Gupta and Sah, 2008). Therefore there is a strong need to implement proper techniques to identify and mitigate landslide hazard.

✉ Vikram Gupta
vgupta_wihg@yahoo.com

1 Wadia Institute of Himalayan Geology, Dehradun, Uttarakhand, India

2 Department of Geophysics, Kurukshetra University, Kurukshetra, Haryana, India

3 Geology Department, Sikkim University, Gangtok, India

4 Department of Mining Engineering, Indian Institute of Technology (Indian School of Mines), Dhanbad, India

Mathematics and Computers in Simulation

Volume 222, March 2024, Pages 259-268

Original article

Modeling the effect of non-pharmaceutical measures and vaccination on the spread of two variants of COVID-19 in India

Anil K. Vaishnavi, Kamal Bisoyi


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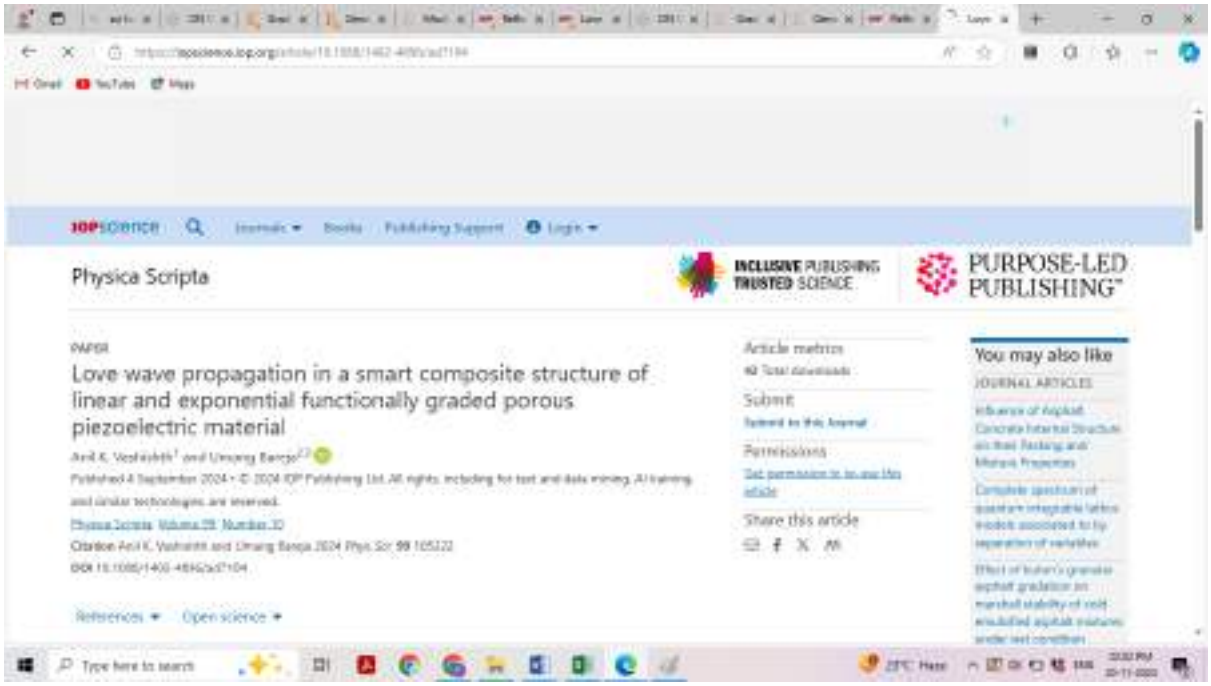
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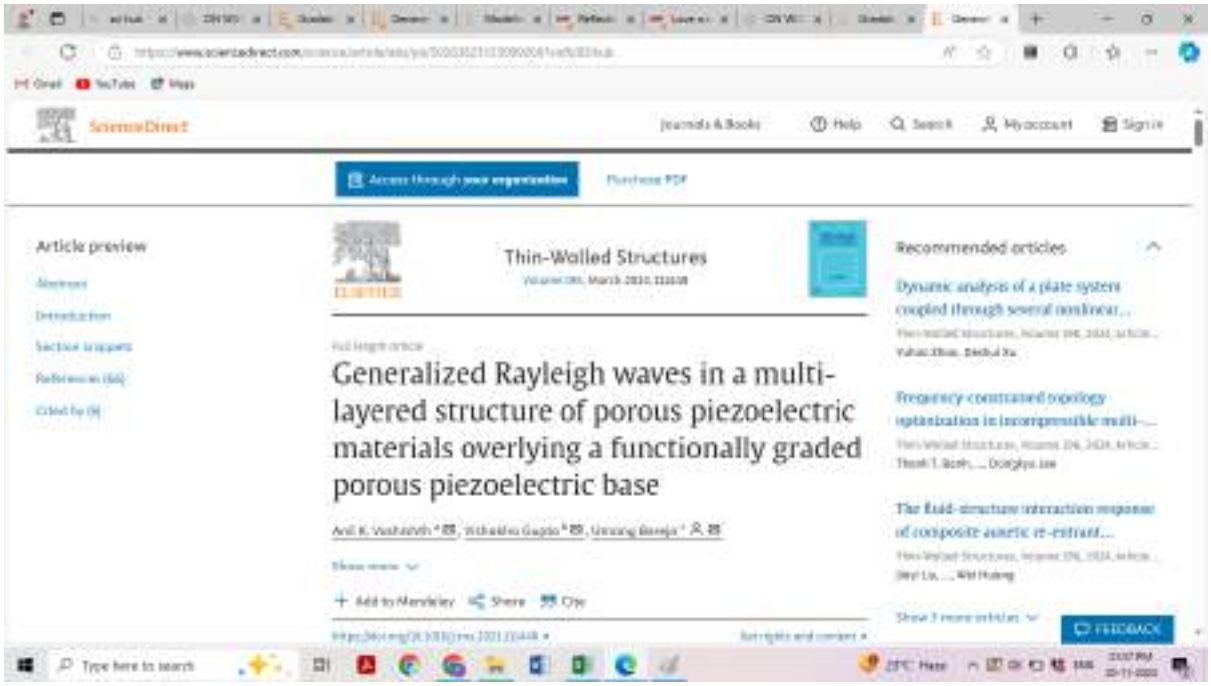
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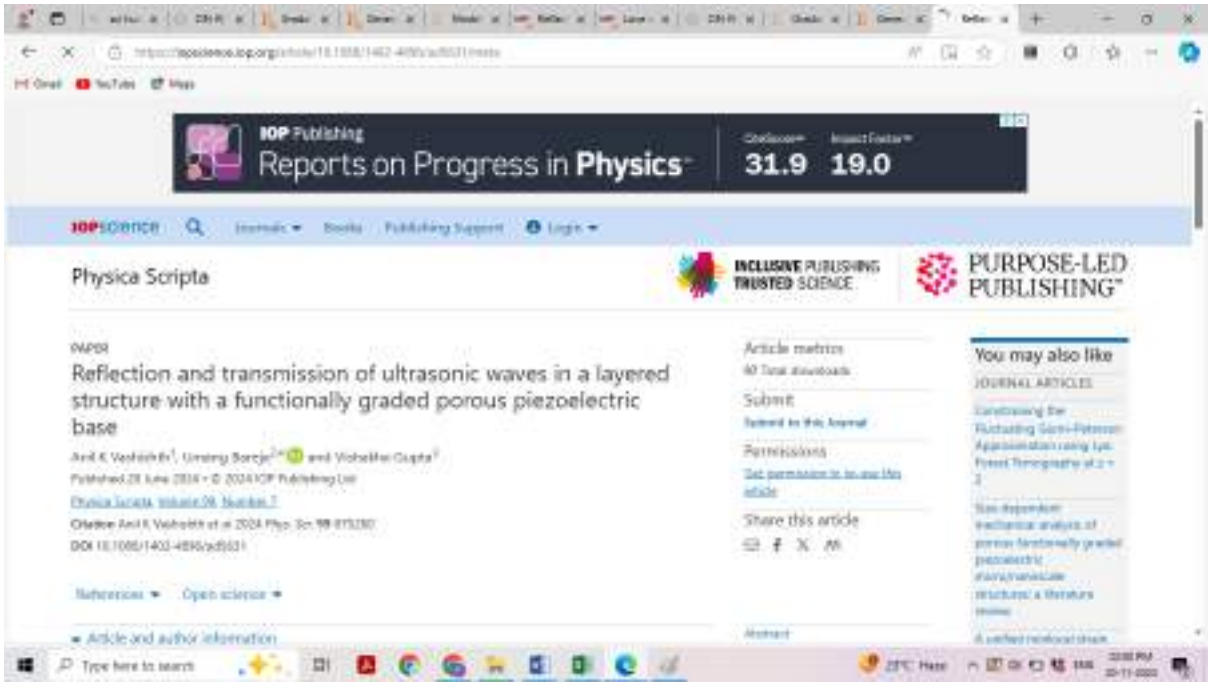
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Ser. Math. Inform. Vol. 38, No 4 (2023), 671–681
https://doi.org/10.22190/FUMI211022043Y
Original Scientific Paper

ON WIJSMAN DEFERRED STATISTICAL CONVERGENCE OF DOUBLE SEQUENCES OF SETS

Mehmet Çağri Yilmazer[1], Mikail Et[1], Vinod K. Bhardwaj[2] and Sandeep Gupta[3]

[1]**Firat University, Faculty of Science**
Department of Mathematics, 23119 Elazıg, Turkey
[2] **Kurukshetra University, Faculty of Science**
Department of Mathematics, 136119 Kurukshetra, India
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Abstract. In this article, we introduce the concepts of Wijsman deferred statistical convergence and Wijsman strong deferred Cesaro summability for double sequences of sets. Additionally, some properties and based results have been established under a few restrictions.
Keywords: statistical convergence, Cesaro summability, double sequences of sets.

1. Introduction

The idea of statistical convergence was given by Zygmund [34] in the first edition of his monograph published in Warsaw in 1935. The concept of statistical convergence was introduced by Steinhaus [31] and Fast [14] and later reintroduced by Schoenberg [30] independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, Ergodic theory, Number theory, Measure theory, Trigonometric series, Turnpike theory and Banach spaces. Later on it was further investigated from the sequence space point of view and linked with summability theory by Altın et al. [3], Bhardwaj et al. ([5],[6],[7]), Cakalli [8], Caserta et al. [9], Connor [10], Dagadur and Sezgek [11],

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Nuray et al. ([23],[24],[25]), Et et al. ([12],[13],[16],[17],[28]), Fridy [15], Işık and Akbaş ([4],[18],[19]), Küçükaslan and Yılmaztürk [20], Mursaleen et al. ([21, 22]), Salat [27], Savas [29] and many others.

Agnew [1] introduced the concept of deferred Cesaro mean of real (or complex) valued sequences $x = (x_k)$ defined by

$$(D_p^q(x))_n = \frac{1}{q_n - p_n} \sum_{k=p_n+1}^{q_n} x_k, n = 1, 2, 3, ...$$

where $p = (p_n)$ and $q = (q_n)$ are two sequences of non-negative integers satisfying

$$p_n < q_n \; and \; \lim_{n\to\infty} q_n = \infty.$$

A sequence $x = (x_k)$ is said to be deferred statistically convergent to L provided that

$$\lim_{n\to\infty} \frac{|\{p_n < k \leq q_n : |x_k - L| \geq \varepsilon\}|}{q_n - p_n} = 0$$

for each $\varepsilon > 0$ and it is written by $S_p^q - \lim x_k = L$ [20].

Let (X, ρ) be a metric space. The distance $d(x, A)$ from a point x to a non-empty subset A of (X, ρ) is defined to be

$$d(x, A) = \inf_{y\in A} \rho(x, y)$$

If $\sup_k d(x, A_k) < \infty$ (for each $x \in X$), then we say that the sequence $\{A_k\}$ is bounded.

A set of sequence $\{A_k\}$ is said to be Wijsman statistical convergent to A provided that

$$d(x, A) = \lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |d(x, A_k) - d(x, A)| \geq \varepsilon\}| = 0$$

if for each $\varepsilon > 0$ and for each $x \in X$. It is written by $st - \lim_W A_k = A$.

By the convergence of a double sequence we mean the convergence in Pringsheim's sense [26]. A double sequence $x = (x_{kj})_{k,j\in\mathbb{N}}$ of real numbers is said to be convergent to $L \in \mathbb{R}$ in Pringsheim's sense if for any $\varepsilon > 0$, there exists $N_\varepsilon \in \mathbb{N}$ such that $|x_{kj} - L| < \varepsilon$, whenever $k, j > N_\varepsilon$. In this case we write $P - \lim_{k,j\to\infty} x_{kj} = L$ or $\lim_{k,j\to\infty} x_{kj} = L$.

A double sequence $x = (x_{kj})$ of real numbers is called to be bounded if there exists a positive real number M such that $|x_{kj}| < M$, for all $k, j \in \mathbb{N}$. In other words $\|x\|_\infty = \sup_{k,j} |x_{kj}| < \infty$.

A double sequence $x = (x_{kj})$ is said to be statistically convergent to L provided that

$$\lim_{m,n\to\infty} \frac{1}{mn} |\{(k, j) : k \leq n, j \leq m : |x_{kj} - L| \geq \varepsilon\}| = 0,$$

Many worthwhile developments of double sequences in summability methods can be found in ([2],[11],[21],[22],[24],[25],[29],[32],[33]).

2. Main Results

In this section, Wijsman deferred statistical convergence and Wijsman strongly deferred Cesàro convergence of double sequences of sets will be defined and the relationship between them will be scrutinized.

Throughout this paper, we will suppose $p = (p_n)$, $q = (q_n), r = (r_m)$ and $t = (t_m)$ are sequences of non-negative integers satisfying the following condition:

$$p_n < q_n,\ t_m < r_m \text{ and } \lim_{n\to\infty} q_n = \infty,\ \lim_{m\to\infty} r_m = \infty \tag{2.1}$$

and

$$\psi_n = q_n - p_n,\ \omega_m = r_m - t_m, D = (p, q, ; r, t) \tag{2.2}$$

Definition 2.1. [11] Let $(p_n), (q_n), (r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions (2.1) and (2.2). The deferred double natural density of any subset S of $\mathbb{N} \times \mathbb{N}$ is denoted by $\delta_D(S)$ and defined as

$$\delta_D(S) = \lim_{m,n\to\infty} \frac{\left|\overline{S}_{nm}\right|}{\psi_n\omega_m},$$

provided the limit exists, where $\overline{S}_{nm} = \{(k, j) \in S : p_n < k \leq q_n \text{ and } t_m < j \leq r_m\}$.

It is obvious that the deferred double natural density of any finte subset of $\mathbb{N}\times\mathbb{N}$ is zero and $\delta_D(S) + \delta_D(\mathbb{N} \times \mathbb{N} - S) = 1$ for any set $S \subset \mathbb{N} \times \mathbb{N}$.

Before proceeding further, we recall a double sequence (A_{kj}) is Wijsman convergent to A if for each $x \in X$, $P{-}\lim_{k,j\to\infty} d(x, A_{kj}) = d(x, A)$ or $\lim_{k,j\to\infty} d(x, A_{kj}) = d(x, A)$, where the convergence is in Pringsheim's sense.

Definition 2.2. Let (p_n), $(q_n), (r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions (2.1) and (2.2). A double sequence $\{A_{kj}\}$ is said to be Wijsman deferred statistically convergent to A provided that

$$\lim_{m,n\to\infty} \frac{1}{\psi_n\omega_m} \left|\left\{(k, j) \in \overline{S}_{nm} : |d(x, A_{kj}) - d(x, A)| \geq \varepsilon\right\}\right| = 0$$

for each $\varepsilon > 0$ and for each $x \in X$ and it is written by $A_{kj} \to A$ (WS_d^2) or $WS_d^2 - \lim A_{kj} = A$. The set of all Wijsman deferred statistically convergent sequences will be denoted by WS_d^2. If $q_n = n, p_n = 0, r_m = m$ and $t_m = 0$, then we write WS^2 instead of WS_d^2.

Definition 2.3. Let (p_n), $(q_n), (r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions (2.1) and (2.2). A double sequence (A_{kj}) is said to be Wijsman strongly deferred convergent to A provided that

$$\lim_{m,n\to\infty} \frac{1}{\psi_n\omega_m} \sum_{k=p_n+1}^{q_n} \sum_{j=t_m+1}^{r_m} |d\left(x, A_{kj} - d\left(x, A\right)\right)| = 0$$

for each $\varepsilon > 0$ and for each $x \in X$, and it is written by $A_{kj} \to A$ $\left(WN_d^2\right)$ or $WN_d^2 - \lim A_{kj} = A$. The set of all Wijsman strongly deferred convergent sequences will be denoted by WN_d^2. If $q_n = n$, $p_n = 0$, $r_m = m$, $t_m = 0$, then we write WN^2 instead of WN_d^2.

If we take $q_n = k_n$, $p_n = k_{n-1}$, $r_m = j_m$, $t_m = j_{m-1}$, where $\theta = \{(k_n, j_m)\}$ are double lacunary sequences, then WS_d^2-convergence is the same as Wijsman lacunary statistical convergence of double sequences of sets and WN_d^2-convergence coincides with Wijsman lacunary strongly convergent of double sequences of sets [25].

We first show that a double sequence which is Wijsman strongly deferred Cesaro summable is Wijsman deferred statistically convergent. However, the converse is not true, in general.

Theorem 2.1. *Let $(p_n), (q_n), (r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions* (2.1) *and* (2.2). *If $WN_d^2 - \lim A_{kj} = A$, then $WS_d^2 - \lim A_{kj} = A$.*

Proof. We assume that $WN_d^2 - \lim A_{kj} = A$. Then for an arbitrary $\varepsilon > 0$, we have

$$\begin{aligned}
& \frac{1}{\psi_n \omega_m} \sum_{\substack{k=p_n+1 \\ j=t_m+1}}^{q_n, r_m} |d(x, A_{kj}) - d(x, A)| \\
= & \frac{1}{\psi_n \omega_m} \left(\sum_{\substack{k=p_n+1 \\ j=t_m+1 \\ |d(x,A_{kj}) - d(x,A)| \geq \varepsilon}} + \sum_{\substack{k=p_n+1 \\ j=t_m+1 \\ |d(x,A_{kj}) - d(x,A)| < \varepsilon}} \right) |d(x, A_{kj}) - d(x, A)| \\
\geq & \frac{1}{\psi_n \omega_m} \sum_{\substack{k=p_n+1 \\ j=t_m+1 \\ |d(x,A_{kj}) - d(x,A)| \geq \varepsilon}} |d(x, A_{kj}) - d(x, A)| \\
\geq & \varepsilon \frac{\left| \left\{ (k,j) \in \overline{S}_{nm} : |d(x, A_{kj}) - d(x, A)| \geq \varepsilon \right\} \right|}{\psi_n \omega_m}.
\end{aligned}$$

By taking limit as $n, m \to \infty$, we obtain

$$\lim_{m,n \to \infty} \frac{\left| \left\{ (k,j) \in \overline{S}_{nm} : |d(x, A_{kj}) - d(x, A)| \geq \varepsilon \right\} \right|}{\psi_n \omega_m} = 0.$$

The converse of Theorem 2.1. is not true, in general. For this; $q_n = k_n$, $p_n = k_{n-1}$, $r_m = j_m$, $t_m = j_{m-1}$, where $\theta = \{(k_n, j_m)\}$ are double lacunary sequences and define a sequence $\{A_{kj}\}$ as follows:

$$A_{kj} = \begin{cases} \{(k,j)\}, & \text{if } k_{n-1} < k \leq k_{n-1} + \left[\sqrt{h_n}\right], j_{m-1} < j \leq j_{m-1} + \left[\sqrt{h_m}\right] \\ & (n, m = 1, 2, ...) \\ \{(0,0)\}, & \text{otherwise.} \end{cases}.$$

Note that $\{A_{kj}\}$ is not bounded. For every $\varepsilon > 0$ and for each $x \in X$, we get

$$\begin{aligned} & \frac{1}{h_n h_m} \left|\{(k,j) \in I_{nm} : |d(x, A_{kj}) - d(x, \{(0,0)\})| \geq \varepsilon\}\right| \\ = \quad & \frac{\left[\sqrt{h_n}\right]\left[\sqrt{h_m}\right]}{h_n h_m} \to 0 \text{ as } n, m \to \infty, \end{aligned}$$

that is, $A_{kj} \to \{(0,0)\}\,(WS_d^2)$. But

$$\begin{aligned} & \frac{1}{h_n h_m} \sum_{(k,j)\in I_{nm}} |d(x, A_{kj}) - d(x, \{(0,0)\})| \\ = \quad & \frac{1}{h_n h_m}\left[\frac{\left(\left[\sqrt{h_n}\right]\left(\left[\sqrt{h_n}\right]+1\right)\right)\left(\left[\sqrt{h_m}\right]\left(\left[\sqrt{h_m}\right]+1\right)\right)}{4}\right] \\ \to \quad & \frac{1}{4} \end{aligned}$$

Therefore, $A_{kj} \nrightarrow \{(0,0)\}\,(WN_d^2)$. $\square$

The following theorem establishes that for bounded double sequences $\{A_{kj}\}$, the converse of Theorem 2.1. is also true.

Theorem 2.2. *Let (p_n), (q_n), (r_m) and (t_m) be sequences of non-negative integers satisfying the conditions* (2.1), (2.2) *and let $\{A_{kj}\}$ be a bounded double sequence. If $WS_d^2 - \lim A_{kj} = A$, then $WN_d^2 - \lim A_{kj} = A$.*

Proof. Suppose that $\{A_{kj}\}$ is bounded and $WS_d^2 - \lim A_{kj} = A$. In this case, there exists a real number $M > 0$ such that

$$|d(x, A_{kj}) - d(x, A)| \leq M$$

for all $k, j \in \mathbb{N}$. For an arbitrary $\varepsilon > 0$, we have

$$\begin{aligned} & \frac{1}{\psi_n \omega_m} \sum_{\substack{k=p_n+1 \\ j=t_m+1}}^{q_n, r_m} |d(x, A_{kj}) - d(x, A)| \\ = \quad & \frac{1}{\psi_n \omega_m}\left(\sum_{\substack{k=p_n+1 \\ j=t_m+1 \\ |d(x,A_{kj})-d(x,A)|\geq\varepsilon}} |d(x, A_{kj}) - d(x, A)|\right) \\ & + \frac{1}{\psi_n \omega_m}\left(\sum_{\substack{k=p_n+1 \\ j=t_m+1 \\ |d(x,A_{kj})-d(x,A)|<\varepsilon}} |d(x, A_{kj}) - d(x, A)|\right) \\ \leq \quad & M \frac{\left|\{(k,j) \in \overline{S}_{nm} : |d(x, A_{kj}) - d(x, A)| \geq \varepsilon\}\right|}{\psi_n \omega_m} + \varepsilon \end{aligned}$$

Since

$$\lim_{m,n\to\infty} \frac{\left|\left\{(k,j)\in \overline{S}_{nm} : |d(x,A_{kj})-d(x,A)|\geq \varepsilon\right\}\right|}{\psi_n\omega_m} = 0 \text{ as } n,m\to\infty$$

we get

$$\lim_{m,n\to\infty} \frac{1}{\psi_n\omega_m} \sum_{\substack{k=p_n+1\\ j=t_m+1}}^{q_n,r_m} |d(x,A_{kj})-d(x,A)| = 0 \text{ as } n,m\to\infty.$$

□

Theorem 2.3. *Let (p_n), $(q_n),(r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions* (2.1) *and* (2.2)*. A Wijsman convergent double sequence (A_{kj}) is Wijsman deferred statistically convergent, but converse need not be true.*

Proof. The proof follows in view of the fact that the deferred double natural density of any finite set is zero. However, the converse is not true, in general. Example in Theorem 2.1. provides a double sequence (A_{kj}) of sets which is Wijsman deferred statistically convergent but fails to be Wijsman convergent. □

We next show that the under certain condition Wijsman statistically convergent double sequences are Wijsman deferred statistically convergent.

Theorem 2.4. *Let (p_n), $(q_n),(r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions* (2.1), (2.2) *and* $\lim_{n,m\to\infty} \frac{nm}{\psi_n\omega_m} = a > 0$*. If $WS^2 - \lim_W A_{kj} = A$, then $WS_d^2 - \lim A_{kj} = A$.*

Proof. If $WS^2 - \lim_W A_{kj} = A$, then we have

$$\lim_{n,m\to\infty} \frac{1}{nm} \left|\{(k,j) : k\leq n, j\leq m : |d(x,A_{kj})-d(x,A)|\geq\varepsilon\}\right| = 0.$$

Since

$$\begin{aligned} &\left\{(k,j)\in \overline{S}_{nm}:\ |d(x,A_{kj})-d(x,A)|\geq\varepsilon\right\} \\ \subset\ &\{(k,j) : k\leq n,\ j\leq m,\ |d(x,A_{kj})-d(x,A)|\geq\varepsilon\} \end{aligned}$$

we have

$$\begin{aligned} &\left|\left\{(k,j)\in \overline{S}_{nm}:\ |d(x,A_{kj})-d(x,A)|\geq\varepsilon\right\}\right| \\ \leq\ &\left|\{(k,j) : k\leq n,\ j\leq m,\ |d(x,A_{kj})-d(x,A)|\geq\varepsilon\}\right| \end{aligned}$$

and so

$$\begin{aligned} &\frac{\left|\left\{(k,j)\in \overline{S}_{nm}:\ |d(x,A_{kj})-d(x,A)|\geq\varepsilon\right\}\right|}{\psi_n\omega_m} \\ \leq\ &\frac{nm}{\psi_n\omega_m} \frac{\left|\{(k,j) : k\leq n,\ j\leq m : |d(x,A_{kj})-d(x,A)|\geq\varepsilon\}\right|}{nm} \end{aligned}$$

Hence

$$\lim_{n,m\to\infty} \frac{\left|\left\{(k,j)\in \overline{S}_{nm}:\ |d(x,A_{kj})-d(x,A)|\geq \varepsilon\right\}\right|}{\psi_n\omega_m}=0$$

that is $WS_d^2-\lim A_{kj}=A$. $\square$

In the next theorem, we arrive at the same result as established in Theorem 4, but under a different condition.

Theorem 2.5. *Let (p_n), $(q_n),(r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions* (2.1), (2.2) *and $\{A_{kj}\}$ be double sequence of sets. If $\liminf_n \frac{q_n}{p_n}>1$ and $\liminf_m \frac{r_m}{t_m}>1$, then $WS^2-\lim_W A_{kj}=A$ implies $WS_d^2-\lim A_{kj}=A$.*

Proof. Assume that $\liminf_n \frac{q_n}{p_n}>1$ and $\liminf_m \frac{r_m}{t_m}>1$, then there exist $\alpha,\beta>0$ such that $\frac{q_n}{p_n}\geq 1+\alpha$ and $\frac{r_m}{t_m}\geq 1+\beta$ for sufficiently large n,m which implies that

$$\begin{aligned}
\frac{q_n}{p_n} &\geq 1+\alpha \Rightarrow \frac{q_n-p_n}{q_n}\geq \frac{\alpha}{1+\alpha}\\
\frac{r_m}{t_m} &\geq 1+\beta \Rightarrow \frac{r_m-t_m}{r_m}\geq \frac{\beta}{1+\beta}\\
\frac{(q_n-p_n)}{q_n}\frac{(r_m-t_m)}{r_m} &\geq \frac{\alpha\beta}{(1+\alpha)(1+\beta)}\\
&\Rightarrow \frac{\psi_n}{q_n}\frac{\omega_m}{r_m}\geq \frac{\alpha\beta}{(1+\alpha)(1+\beta)}
\end{aligned}$$

If $WS^2-\lim_W A_{kj}=A$, then for every $\varepsilon>0$ and for sufficiently larger n,m we get

$$\begin{aligned}
&\frac{1}{q_nr_m}\left|\{(k,j):k\leq q_n,\ j\leq r_m:|d(x,A_{kj})-d(x,A)|\geq\varepsilon\}\right|\\
\geq\ &\frac{1}{q_nr_m}\left|\left\{(k,j)\in\overline{S}_{nm}:|d(x,A_{kj})-d(x,A)|\geq\varepsilon\right\}\right|\\
\geq\ &\frac{\alpha\beta}{(1+\alpha)(1+\beta)}\left(\frac{1}{\psi_n\omega_m}\left|\left\{(k,j)\in\overline{S}_{nm}:\ |d(x,A_{kj})-d(x,A)|\geq\varepsilon\right\}\right|\right)
\end{aligned}$$

for each $x\in X$. Hence, $WS_d^2-\lim A_{kj}=A$. $\square$

Following the same technique , as that of Lemma 1,1 of Salat [27], we have

Theorem 2.6. *Let (p_n), $(q_n),(r_m)$ and (t_m) be sequences of non-negative integers satisfying the conditions* (2.1) *and* (2.2). *A double sequence $\{A_{kj}\}$ of sets is Wijsman deferred statistically convergent to A if and only if there exists a set $K\subset\mathbb{N}\times\mathbb{N}$ such that $\delta_D(K)=1$ and $\lim_{\substack{k,j\to\infty\\(k,j)\in K}} A_{kj}=A$.*

Proof. For $r \in \mathbb{N}$, let $K^r = \left\{(k,j) \in \mathbb{N} \times \mathbb{N} : |d(x, A_{kj}) - d(x, A)| < \frac{1}{r}\right\}$. As $K^r = \mathbb{N} \times \mathbb{N} - \left\{(k,j) \in \mathbb{N} \times \mathbb{N} : |d(x, A_{kj}) - d(x, A)| \geq \frac{1}{r}\right\}$ and $WS_d^2 - \lim A_{kj} = A$ so $\delta_D(K^r) = 1$. As

$$\begin{aligned} & \left\{(k,j) \in \mathbb{N} \times \mathbb{N} : |d(x, A_{kj}) - d(x, A)| < \frac{1}{r+1}\right\} \\ \subset & \left\{(k,j) \in \mathbb{N} \times \mathbb{N} : |d(x, A_{kj}) - d(x, A)| < \frac{1}{r}\right\} \end{aligned}$$

so $K^1 \supset K^2 \supset K^3 \cdots \supset K^r \supset K^{(r+1)} \cdots$ and $\delta_D(K^r) = 1$. Let us choose $(n_1, m_1) \in K^1$. Then there exists $(n_2, m_2) > (n_1, m_1)$, $(n_2, m_2) \in K^2$ such that for all $(n, m) \geq (n_2, m_2)$ we have

$$\frac{1}{\psi_n \omega_m}\left|\left\{(k,j) \in \mathbb{N} \times \mathbb{N} : p_n < k \leq q_n \text{ and } t_m < j \leq r_m \;\; |d(x, A_{kj}) - d(x, A)| < \frac{1}{2}\right\}\right| > \frac{1}{2}.$$

Choose $(n_3, m_3) > (n_2, m_2)$,$(n_3, m_3) \in K^3$ such that for all $(n, m) \geq (n_3, m_3)$ we have

$$\frac{1}{\psi_n \omega_m}\left|\left\{(k,j) \in \mathbb{N} \times \mathbb{N} : p_n < k \leq q_n \text{ and } t_m < j \leq r_m \;\; |d(x, A_{kj}) - d(x, A)| < \frac{1}{3}\right\}\right| > \frac{2}{3}.$$

We continue this process and construct by induction a sequence $(n_1, m_1) < (n_2, m_2) < (n_3, m_3) \cdots (n_j, m_j) < \cdots$ such that $(n_j, m_j) \in K^j$ with

$$\frac{1}{\psi_n \omega_m}\left|\left\{(k,j) \in \mathbb{N} \times \mathbb{N} : p_n < k \leq q_n \text{ and } t_m < j \leq r_m \;\; |d(x, A_{kj}) - d(x, A)| < \frac{1}{j}\right\}\right| > \frac{j-1}{j}$$

for all $(n, m) \geq (n_j, m_j)$.

Let us consider $K = \Big([1, n_1) \times [1, m_1)\Big) \bigcup_j \left(\Big([\ n_j, n_{j+1}) \times [m_j, m_{j+1})\Big) \bigcap K^j\right)$.

Now for each (n, m) such that $(n_j, m_j) < (n, m) < (n_{j+1}, m_{j+1})$, we get

$$\begin{aligned} & \frac{1}{\psi_n \omega_m} |\{(k,j) \in K : p_n < k \leq q_n \text{ and } t_m < j \leq r_m\}| \\ \geq & \frac{1}{\psi_n \omega_m}\left|\left\{(k,j) \in \mathbb{N} \times \mathbb{N} : p_n < k \leq q_n \text{ and } t_m < j \leq r_m \;\; |d(x, A_{kj}) - d(x, A)| < \frac{1}{j}\right\}\right| \\ > & \frac{j-1}{j} \text{ for each j} \in \mathbb{N}. \end{aligned}$$

From this, we have $\delta_D(K) = 1$. Let $\varepsilon > 0$. Choose j such that $\frac{1}{j} < \varepsilon$. Now for all $(n, m) \geq (n_j, m_j)$ and $(n, m) \in K$, choose $p \geq j$ such that $(n_p, m_p) \leq (n, m) \leq (n_{p+1}, m_{p+1})$, we get $(n, m) \in K^p$ which in turn yields

$$|d(x, A_{kj}) - d(x, A)| < \frac{1}{p} \leq \frac{1}{j} < \varepsilon.$$

Conversely, suppose there exists a set $K \subset \mathbb{N} \times \mathbb{N}$ such that $\delta_D(K) = 1$ and $\lim_{\substack{k,j \to \infty \\ (k,j) \in K}} A_{kj} = A$. For $\varepsilon > 0$, there exists $(n_0, m_0) \in \mathbb{N} \times \mathbb{N}$ such that

$|d(x, A_{kj}) - d(x, A)| < \varepsilon$ for all $(k, j) \geq (n_0, m_0)$ and $(k, j) \in K$.
Taking $K_\varepsilon = \{(k, j) \in \mathbb{N} \times \mathbb{N} : |d(x, A_{kj}) - d(x, A)| \geq \varepsilon\}$, the result follows in view of the facts that $K_\varepsilon \subset (\mathbb{N} \times \mathbb{N}) - K$. □

Before proceeding further first we introduce the following notation:

If $A = (A_{kj})$ is a double sequence of sets such that (A_{kj}) satisfies property P for all (k, j), except a set of deferred double natural density zero, then we say $A = (A_{kj})$ satisfies P for "almost all (k, j) deferred double with respect to $D = (p, q : r, t)$" and we abbreviate this by "a.a. (k, j) deferred double w.r.t.D" where $p = (p_n)$, $q = (q_n), r = (r_m)$ and $t = (t_m)$ be sequences of non-negative integers satisfying the conditions (2.1) *and* (2.2).

Finally we establish that the terms of a Wijsman deferred statistically convergent double sequence (A_{kj}) are coincident with those of a Wijsman convergent sequence for a. a. (k, j) deferred double w.r.t. D.

Theorem 2.7. *A double sequence (A_{kj}) of sets is Wijsman deferred statistically convergent if and only if there exists a Wijsman convergent double sequence (B_{kj}) of sets such that $B_{kj} = A_{kj}$ a. a. (k, j) deferred double w.r.t. D.*

Proof. Let (A_{kj}) is Wijsman deferred statistically convergent to A. So for each $\varepsilon > 0$ we have $\delta_D(K) = 0$ where $K = \{(k, j) \in \mathbb{N} \times \mathbb{N} : |d(x, A_{kj}) - d(x, A)| \geq \varepsilon\}$.
Consider

$$B_{kj} = \begin{cases} A_{kj}, & \text{if } (k, j) \in (\mathbb{N} \times \mathbb{N}) - K \\ A, & \text{otherwise} \end{cases}.$$

Then (B_{kj}) is a Wijsman convergent double sequence of sets converging to A such that $B_{kj} = A_{kj}$ a. a. (k, j) deferred double w.r.t. D.
Conversely, let (B_{kj}) is a Wijsman convergent double sequence of sets converging to A such that $B_{kj} = A_{kj}$ a. a. (k, j) deferred double w.r.t. D. Then for $\varepsilon > 0$ there exists $k_0, j_0 \in \mathbb{N}$ such that $|d(x, B_{kj}) - d(x, A)| < \varepsilon$ for all $(k, j) \geq (k_0, j_0)$. Let $K = \{(k, j) \in \mathbb{N} \times \mathbb{N} : B_{kj} \neq A_{kj}\}$. Now $\{(k, j) : |d(x, A_{kj}) - d(x, A)| \geq \varepsilon\} \subset K \cup [1, k_0) \times [1, j_0)$, yields the result. □

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Representation of fundamental solution and vibration of waves in photothermoelastic under MGTE model

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Abstract. In this paper, Moore-Gibson-Thompson theory of thermoelasticity is considered to investigate the fundamental solution and vibration of plane wave in an isotropic photothermoelastic solid. The governing equations are made dimensionless for further investigation. The dimensionless equations are expressed in terms of elementary functions by assuming time harmonic variation of the field variables (displacement, temperature distribution and carrier density distribution). Fundamental solutions are constructed for the system of equations for steady oscillation. Also some preliminary properties of the solution are explored. In the second part, the vibration of plane waves are examined by expressing the governing equation for two dimensional case. It is found that for the non-trivial solution of the equation yield that there exist three longitudinal waves which advance with the distinct speed, and one transverse wave which is free from thermal and carrier density response. The impact of various models (i)Moore-Gibson-Thomson thermoelastic (MGTE)(2019), (ii) Lord and Shulman's (LS)(1967) , (iii) Green and Naghdi type-II(GN-II)(1993) and (iv) Green and Naghdi type-III(GN-III)(1992) on the attributes of waves i.e., phase velocity, attenuation coefficient, specific loss and penetration depth are elaborated by plotting various figures of physical quantities. Various particular cases of interest are also deduced from the present investigations. The results obtained can be used to delineate various semiconductor elements during the coupled thermal, plasma and elastic wave and also find the application in the material and engineering sciences.

Keywords: fundamental solution; Moore-Gibson-Thompson thermoelastic model; photothermoelastic isotropic; plane waves; steady oscillations

1. Introduction

Study of mechanical and thermal interaction within a solid medium is of emended significance in various scientific fields. There are few examples such as high energy particle accelerated devices, modern aeronautical and astronomical engineering and different system exploited in nuclear and industrial applications with the consideration of second sound effect in thermoelastic model plays a significant role in analysing elastic body with in a variety of scientific and technological fields. In contradiction with physical observation the infinite thermal propagation speed is observed through conventional uncoupled theories.The coupled thermoelasticity proposed by Biot (1956) in order to

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eradicate the classic uncoupled principle's inherent paradox. Generalized thermoelasticity theories are designed to solve the weaknesses and shortcomings inherent in classic dynamic thermoelasticity coupled theory. Lord and Shulman (1967) and Green and Lindsay (1972) developed generalized theory of thermoelasticity involving one and two relaxation parameters.

Green and Naghdi (1991, 1992, 1993) derived three models in thermoelasticity which are labelled as GN-I, II and III models. The linearized form of model-I reduces to classical heat conduction theory whereas linearized version of model-II and III permit propagation of thermal waves at finite speed.GN-II (1993) shows a feature which makes it different from other thermoelastic models as it does not allow dissipation of thermal energy. The model GN-III (1992) contains the thermal displacement gradient alongwith temperature gradient among the constitutive variables and admits the dissipation of energy. Tzou (1995) proposed the dual-phase heat conduction law which is a more common one with two different phase delays, one in the heat flow vector and the second in the temperature gradient, which takes into account the effects of the microstructure on the heat transmission mechanism, in order to evaluate the delayed reaction caused by the microstructure effects over time. One of the most recent advances in the theory of thermoelasticity is the three-phase lags suggested by Roychoudhari (2007). This model also has phase delays of thermal displacement gradients, in addition to the phase lags in the hot flux vector and temperature gradient. These two suggestions, involving different derivatives as the Taylor spectrum approaches the heat flow and temperature gradients, assume that the suggestion by Roychoudhari seeks to restore Green and Naghdi models if various Taylor approaches are taken into account.

Abbas and Abd-alla (2008) investigated the thermoelastic interactions in an infinite orthotropic elastic medium with a cylindrical cavity subjected to ramp-type heating applied to the boundary of the cavity.Abbas (2011) discussed the influence of reinforcement on the total deformation body by applying Green and Naghdi theory. Marin *et al.* (2014) studied the basic equations and conditions of the mixed initial boundary value problem in the context of micropolar thermoelastic diffusion, which is an extension of known Saint-Venant's principle from classical Elasticity. Zenkour and Abbas (2014) analysed the nonlinear transient thermal stress of temperature dependent infinite cylinders subjected to a decaying with time thermal loading. Abbas (2015) studied the natural frequencies, thermoelastic damping and frequency shift of a thermoelastic hollow sphere into the context of the generalized thermoelasticity theory with one relaxation time. Abbas *et.al.* (2016) examined the propagation of waves in thermoelastic plate in the context LS theory and obtained an analytical solution for the temperature, displacement components, and stresses using the eigenvalue approach. Abbas and Kumar (2016) studied the plane problem in initially stressed thermoelastic half-space with voids due to thermal source. Ghanmi and Abbas (2019) introduced the bioheat equation under fractional derivatives to study the thermal damage within the skin tissue during the thermal therapy.

The semiconducting materials were used widely in modern engineering, with the development of technologies. The study of wave propagation in a semiconducting medium will have important academic significance and application value. Of recent interest is the relevance of the excitation of short elastic pulses (high-frequency elastic waves) by photothermal means to several areas of applied physics including the photoacoustic microscope, thermal wave imaging, determination of thermoelastic material parameters, non-destructive evaluation of devices, monitoring of laser drilling, and laser annealing and melting phenomena in semiconductors. When a semiconductor surface is exposed to a beam of laser, some electrons will be excited. In this case, the photo-excited free carriers will be produced with non-radiative transitions, and a recombination between electron and hole plasma occurs. Many efforts are made to explore the nature of semiconductors in last few

years. The technique adopted is photo acoustic and photo thermal technology.

Photoacoustic (PA) and photothermal (PT) science and technology have extensively developed new methods in the investigation of semiconductors and microelectronic structures during the last few years. PA and PT techniques were recently established as diagnostic methods with good sensitivity to the dynamics of photoexcited carrier (Mandelis 1987, Almond and Patel 1996, Mandelis and Michaelian 1997, Nikolic and Todorovic 1989). Photogeneration of electron-hole pairs, i.e., the carriers-diffusion wave or plasma wave, generated by an absorbed intensity modulated laser beam, may, play a dominant role in PA and PT experiments for most semiconductor materials. Depth dependent plasma waves contribute to the generation of periodic heat and mechanical vibrations, i.e., thermal and elastic waves. This mechanism of elastic wave generation is a specific of semiconductors. The electronic deformation mechanism is based on the fact that photogenerated plasma in the semiconductor causes deformation of the crystal lattice, i.e., deformation of the potential of the conduction and valence bands in the semiconductor. Thus, photoexcited carries may cause local strain in the sample. This strain in turn may produce plasma waves in the semiconductor in a manner analogous to thermal wave generation by local periodic elastic deformation.

The difference influences of the thermoelastic and electronic deformations in semiconductor media with disregard the coupling between the plasma and the thermoelastic equations have been analyzed by numerous researchers (McDonald and Wetsel 1978, Jackson and Amer 1980, Stearns and Kino 1985). Todorovic (2003a, b, 2005) presented the theoretical analysis to describe two phenomena that provide information about the properties of transport and carrier recombinations in the semiconducting medium. The changes in the propagations of thermal and plasma waves go back to the linear coupling between the thermal and the mass transport (i.e., thermodiffusion) have included. Sharma (2010) investigated the boundary value problems in generalized thermodiffusive elastic medium. Sharma and Sharma (2014) investigated the temperature fluctuations in tissues based on Penne's bio-heat transfer equation. Hobiny and Abbas (2019) investigated the photothermal interactions in a two-dimensional semiconducting half-space under the coupled of thermo-elastic theory and plasma wave based on Green and Naghdi theory. Abbas *et al*. (2020) examined the effect of the variability of thermal conductivity in semi-conductor media with cylindrical cavity using the eigen value methods. Marin *et al.* (2021) analysed a new picture of the porothermoelastic model using the fractional calculus with thermal relaxation times. Sharma and Kumar (2021) developed a dynamic mathematical model of photothermoelastic (semiconductor) medium to analyse the deformation due to inclined loads. Sharma and Kumar (2022) examined photothermoelastic deformation in dual phase lag model due to concentrated inclined load. Kumar *et al.* (2022) investigated deformation due to thermomechanical carrier density loading in orthotropic photothermoelastic plate.

The Moore-Gibson-Thompson theory of thermoelasticity has received immense level of concern in recent years. This theory starting from a third-order differential equation and built in the context of some considerations related to fluid mechanics by Thompson (1972). Quintanilla (2019) presented a Moore-Gibson-Thompson thermoelasticity in which the heat conduction equation is described by MGT equation. This equation is obtained by incorporating relaxation parameter in the GN-III (1992) model. Conti *et al.* (2020) explored thermoelasticity of MGT type with history dependence in the temperature. Conti *et al.* (2020a) presented the analyticity of viscoelastic plate under MGT model of thermoelasticity. Quintanilla (2020) proposed a new thermoelastic model of MGT heat conduction equation with two temperature and examine some basic theorems. Pellicer and Quintanilla (2020) examined the uniqueness and instability of some thermomechanical problems based on MGT theory of thermoelasticity.

Bazarra *et al.* (2020) examined a thermoelastic problem numerically where the heat conduction law is modelled by using Moore-Gibson-Thompson equation. Marin (2020) presented mixed initial-boundary value problem in the context of the Moore-Gibson-Thompson theory of thermoelasticity for dipolar bodies. Abouelregal *et al.* (2021) presented a modified Moore–Gibson–Thompson photo-thermoelastic model for a rotating semiconductor half-space under magnetic field. Kumar *et al.* (2022) studied the deformation due to thermomechanical and carrier density loading in orthotropic photothermoelastic plate under Moore-Gibson-Thompson thermoelastic model. Sharma *et al.* (2013b) studied the wave propagation in anisotropic thermoviscoelastic medium in the context Green-Naghdi theories of type-II and type-III. The concept of fundamental solutions has significant role in investigation of various problem of mathematical physics, which are encountered in many mathematical, mechanical, physical and engineering applications. The applications of fundamental solutions to a recently developed area of boundary value method has provided a corporeal advantage, is that an integral representation of the solution to a boundary value problem (BVP) in terms of fundamental solution can be solved more easily by numerical methods with respect to differential equation having specific boundary and initial conditions. Several methods are known for constructing fundamental solutions of the system of differential equations, theory of elasticity and thermoelasticity, which are given in the books [Kupradze (1979), Nowacki (1962,1975)]. For a historical and bibliographical material on the fundamental solutions of partial differential equation is also available in the books [Hörmander (1983), Kythe (1996).]

Sharma *et al.* (2013a) investigated the propagation of plane waves and fundamental solution in a homogeneous isotropic electro-microstretch elastic solids. Sharma *et al.* (2014) investigated the propagation of plane waves and fundamental solution of homogeneous isotropic electro-microstretch viscoelastic solids. Svanadze (2017) constructed the fundamental solution and uniqueness theorems in the linear theory of thermoviscoelasticity for solids with double porosity. Kumar *et al.* (2020) constructed the fundamental solution of the system of differential equations in bio-thermoelasticity with dual phase lag in case of steady oscillations. Kumar *et al.* (2021) constructed the basic theorem in terms of elementary function which analyse the behaviour of non-local and dual phase lag model and determine the existence of longitudinal and transverse wave. El-Bary and Atef (2021) obtained the fundamental solution of generalized magneto thermo viscoelasticity with two relaxation times for perfect isotropic conduction. Kumar and Batra (2022) investigated the fundamental solution and propagation of plane waves in swelling porous thermoelastic medium involving mixtures of solid, fluid, and gas.

In this paper, the fundamental solution and propagation of plane waves in photothermoelastic under Moore-Gibson-Thompson model has been studied. The representation of fundamental solution of system of equations in the case of study oscillations is considered in terms of elementary functions. Some basic properties of the fundamental solution are also established. The phase velocity, attenuation coefficient, specific loss and penetration depth of plane waves for MGTE (2019), LS (1967), GN-II (1993) and GN-III (1992) models are computed and presented graphically with respect to frequency.

2. Basic equations

Let $x=(x_1,x_2,x_3)$ be the point of the Euclidean three- dimensional space E^3.

$|x| = \left(x_1^2 + x_2^2 + x_3^2\right)^{1/2}$, $D_x = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$ and let t denote the time variable.

Following (Todorovic 2003b, Quintanilla 2019), the basic equations for homogeneous isotropic photothermoelastic based on Moore-Gibson-Thompson heat equation in absence of body force, heat source and carrier photogeneration sources are

$$(\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \gamma_t T_{,i} - \gamma_n N_{,i} = \rho \ddot{u}_i \,, \tag{1}$$

$$K\dot{T}_{,ii} + K^* T_{,ii} = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left(\rho C_e \ddot{T} + T_o \gamma_t \ddot{e}_{kk} - \frac{E_g}{\tau}\frac{\partial N}{\partial t}\right), \tag{2}$$

$$D_e N_{,ij} - \frac{\partial N}{\partial t} - \frac{N}{\tau} + \zeta \frac{T}{\tau} = 0. \qquad \text{(i, j, k = 1, 2, 3)} \tag{3}$$

where

λ *and* μ are Lame's constants, T- the temperature distribution, T_o the reference temperature, u_i components of displacement, ρ - the medium density, K thermal conductivity, K^*thermal conductivity rate, D_e the coefficients of carrier diffusion, C_e the specific heat, $N = n - n_o$, n_o equilibrium carrier concentration, E_g the semiconductor energy gap, $\gamma_n = (3\lambda + 2\mu)\alpha_n, \alpha_n$ is coefficient of electronic deformation, $\gamma_t = (3\lambda + 2\mu)\alpha_t, \alpha_t$ is the linear thermal expansion coefficient. $\zeta = \frac{\partial n_o}{\partial T}$ the thermal activation coupling parameter, τ_o the thermal relaxation time, τ - the photogenerated carrier lifetime, t - the time variable.

Following dimensionless parameters are taken as

$$\left(x_1', x_2', x_3', u_1', u_2', u_3'\right) = \eta_1 C_o \left(x_1, x_2, x_3, u_1, u_2, u_3\right) \,, \quad \left(t', \tau_o', \tau'\right) = \eta_1 C_o^2 \left(t, \tau_o, \tau\right), \; T' = \frac{\gamma_t T}{\rho C_o^2}, N' = \frac{N}{n_o} \tag{4}$$

where

$$\eta_1 = \frac{\rho C_e}{K} \,, \; C_o^2 = \frac{\lambda + 2\mu}{\rho}$$

Eqs. (1)-(3) by considering Eq. (4) take the form (after removing primes)

$$g_1 grad\, div\, \boldsymbol{u} + g_2 \Delta \boldsymbol{u} - grad\, T - g_3\, grad\, N = \ddot{\boldsymbol{u}}, \tag{5}$$

$$\Delta \dot{T} + g_4 \Delta T = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left[\ddot{T} + g_5\, div\, \ddot{\boldsymbol{u}} - \frac{g_6}{\tau}\dot{N}\right], \tag{6}$$

$$g_8 \frac{T}{\tau} + \Delta N - g_7 \dot{N} - g_7 \frac{N}{\tau} = 0, \tag{7}$$

where

$$g_1 = \frac{\lambda+\mu}{\lambda+2\mu}, g_2 = \frac{\mu}{\lambda+2\mu}, g_3 = \frac{\gamma_n n_o}{\lambda+2\mu},$$
$$g_4 = \frac{K^*}{K\eta_1 C_o^2}, g_5 = \frac{T_o\gamma_t^2}{K\eta_1 C_o^2\rho}, g_6 = \frac{E_g n_o \gamma_t}{K\eta_1\rho C_o^2},$$
$$g_7 = \frac{1}{\eta_1 D_e}, g_8 = \frac{\zeta\rho C_o^2}{\gamma_t D_e n_o \eta_1}.$$

3. Steady oscillation

For the case of steady oscillation, we assume the displacement vector, temperature distribution and carrier density distribution as

$$\left(\boldsymbol{u}(\boldsymbol{x},t), T(\boldsymbol{x},t), N(\boldsymbol{x},t)\right) = \mathrm{Re}\left[\left(\boldsymbol{u}, T, N\right)e^{-i\omega t}\right] \tag{8}$$

where ω is oscillation frequency and $\omega > 0$.

Using Eq. (8) into Eqs. (5)-(7), reduce the system of equation of steady oscillations as

$$g_1 \, grad\, div\, \boldsymbol{u} + \left(g_2\Delta + \omega^2\right)\boldsymbol{u} - grad\, T - g_3\, grad\, N = 0\,, \tag{9}$$

$$g_{10}\, div\, \boldsymbol{u} + \left(g_{11} + g_{12}\Delta\right)T + \frac{g_{13}}{\tau}N = 0 \tag{10}$$

$$\frac{g_8}{\tau}T + \left(\Delta + \frac{g_{14}}{\tau}\right)N = 0, \tag{11}$$

where

$g_9 = 1 - i\omega\tau_o$, $g_{10} = \omega^2 g_5 g_9$, $g_{11} = \omega^2 g_9$, $g_{12} = -i\omega + g_4$, $g_{13} = -i\omega g_6 g_9$,
$g_{14} = (i\tau\omega - 1)g_7$.

Introducing the matrix differential operator

$$\boldsymbol{F}(\boldsymbol{D}_x) = \left\|\boldsymbol{F}_{gh}(\boldsymbol{D}_x)\right\|_{5\times 5}, \tag{12}$$

where

$$\boldsymbol{F}_{mn}(\boldsymbol{D}_x) = \left(g_2\Delta + \omega^2\right)\delta_{mn} + g_1\frac{\partial^2}{\partial x_m \partial x_n}, \qquad \boldsymbol{F}_{m4}(\boldsymbol{D}_x) = -\frac{\partial}{\partial x_m}, \boldsymbol{F}_{4n}(\boldsymbol{D}_x) = g_{10}\frac{\partial}{\partial x_n},$$
$$\boldsymbol{F}_{44}(\boldsymbol{D}_x) = g_{11} + g_{12}\Delta$$

$$\boldsymbol{F}_{55}(\boldsymbol{D}_x) = \Delta + \frac{g_{14}}{\tau}, \boldsymbol{F}_{45}(\boldsymbol{D}_x) = \frac{g_{13}}{\tau},$$

$$\boldsymbol{F}_{54}(\boldsymbol{D}_x) = \frac{g_8}{\tau}.$$

δ_{mn} is kronecker delta function.

The system of Eqs. (9)-(11) can be rewritten as

$$\boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{U}(\boldsymbol{x}) = 0, \tag{13}$$

where

$\boldsymbol{U} = (\boldsymbol{u}, T, N)$ is a five components vector function on E^3.

we assume that

$$g_2 g_{12} \neq 0. \tag{14}$$

Definition. The fundamental solution of the system of Eqs. (9)-(11) (the fundamental matrix of operator $\boldsymbol{F}$) is the matrix $\boldsymbol{G}(\boldsymbol{x}) = \|G_{gh}(\boldsymbol{x})\|_{5\times5}$ satisfying condition (Hörmandertal 1963)

$$\boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{G}(\boldsymbol{x}) = \delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}) \tag{15}$$

where δ is the Dirac delta, $\boldsymbol{I} = \|\delta_{gh}\|_{5\times5}$ is the unit matrix and $\boldsymbol{x} \in E^3$.

Now we construct $\boldsymbol{G}(\boldsymbol{x})$ in terms of elementary functions.

4. Representation of fundamental solutions

We consider the system of equations

$$g_1 grad\ div\ \boldsymbol{u} + (g_2\Delta + \omega^2)\boldsymbol{u} + g_{10} grad\, T = \boldsymbol{H}, \tag{16}$$

$$-div\,\boldsymbol{u} + (g_{11} + g_{12}\Delta)T + \frac{g_8}{\tau} N = L, \tag{17}$$

$$-g_3\, div\,\boldsymbol{u} + \frac{g_{13}}{\tau} T + \left(\Delta + \frac{g_{14}}{\tau}\right) N = M. \tag{18}$$

where $\boldsymbol{H}$ in Eq. (16) are two vector function on E^3 and L & M are scalar functions on E^3.

The system of Eqs. (16)-(18) can be written in the form

$$\boldsymbol{F}^{tr}(\boldsymbol{D}_x)\boldsymbol{U}(\boldsymbol{x}) = \boldsymbol{Q}(\boldsymbol{x}), \tag{19}$$

where $\boldsymbol{F}^{tr}$ is the transpose of matrix $\boldsymbol{F}$, $\boldsymbol{Q} = (\boldsymbol{H}, L, M)$ and $\boldsymbol{x} \in E^3$.

Applying the operator div to Eq. (16), we obtain

$$(\Delta + \omega^2)div\,\boldsymbol{u} + g_{10}\Delta T = div\,\boldsymbol{H}, \tag{20}$$

Eqs. (20), (17) and (18) may be written in the form

$$N(\Delta)\boldsymbol{S} = \boldsymbol{\psi}, \tag{21}$$

where

$\boldsymbol{S} = (div\boldsymbol{u}, T, N)$ *and* $\boldsymbol{\psi} = (\psi_1, \psi_2, \psi_3) = (divH, L, M)$

and

$$N(\Delta) = \left\| N_{mn} \right\|_{3\times 3} = \begin{Vmatrix} \Delta + \omega^2 & g_{10}\Delta & 0 \\ -1 & g_{11} + g_{12}\Delta & \frac{g_8}{\tau} \\ -g_3 & \frac{g_{13}}{\tau} & \Delta + \frac{g_{14}}{\tau} \end{Vmatrix}_{3\times 3}, \tag{22}$$

Eq. (21) implies

$$\Gamma_1(\Delta)\boldsymbol{S} = \tilde{\boldsymbol{\psi}} \tag{23}$$

also $\tilde{\boldsymbol{\psi}} = (\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_3)$ and $\tilde{\psi}_n = \frac{1}{g_{12}} \sum_{m=1}^{3} N^*_{mn} \psi_m$, $\Gamma_1(\Delta) = \frac{1}{g_{12}} \det N(\Delta)$; $n = 1,2,3$. N^*_{mn} is the cofactor of the elements N_{mn} of the matrix N.

From Eqs. (21) and (23), we notice that

$$\Gamma_1(\Delta) = \prod_{m=1}^{3} (\Delta + \lambda_m^2) , \tag{24}$$

where λ_m^2 , m=1,2,3 are the roots of the equation $\Gamma_1(\Delta)$ or $\Gamma_1(-\kappa)$=0 (w.r.t. κ)

Now applying the operator $\Gamma_1(\Delta)$ to Eq. (16), yield

$$\Gamma_1(\Delta)\left(g_2\Delta + \omega^2\right)\boldsymbol{u} = \Gamma_1(\Delta)\left(-g_1 grad\, div\ \boldsymbol{u} - g_{10} grad\, T + \boldsymbol{H}\right),$$

$$\Gamma_1(\Delta)\left(g_2\Delta + \omega^2\right)\boldsymbol{u} = -g_1 grad\, \psi_1 - g_{10} grad\ \psi_2 + \Gamma_1(\Delta)\boldsymbol{H}, \tag{25}$$

Eq. (25) can be written as

$$\Gamma_1(\Delta)\Gamma_2(\Delta)\boldsymbol{u} = \boldsymbol{\psi}^* \tag{26}$$

where

$$\Gamma_2(\Delta) = \frac{1}{g_2} \det \begin{Vmatrix} \Delta & -\frac{\omega}{g_2} \\ \omega & 1 \end{Vmatrix}_{2\times 2}, \tag{27}$$

and

$$\boldsymbol{\psi}^* = \frac{1}{g_2}\left\{-g_1 grad\, \psi_1 - g_{10} grad\ \psi_2 + \Gamma_1(\Delta)\boldsymbol{H}\right\}, \tag{28}$$

It can be seen that

$$\Gamma_2(\Delta)=(\Delta+\lambda_4^2)$$

where λ_4^2 is a root of the equation $\Gamma_2(-\kappa)=0$ (w.r.t. κ)

On the basis of Eqs. (21) and (26), we obtain

$$\boldsymbol{\Theta}(\Delta)\boldsymbol{U}(\boldsymbol{x})=\hat{\boldsymbol{\psi}}(\boldsymbol{x}), \tag{29}$$

where

$$\hat{\boldsymbol{\psi}}(x)=(\boldsymbol{\psi}^*,\tilde{\psi}_2,\tilde{\psi}_3),$$
$$\boldsymbol{\Theta}(\Delta)=\left\|\boldsymbol{\Theta}_{gh}(\Delta)\right\|_{5\times5},$$
$$\boldsymbol{\Theta}_{mm}(\Delta)=\Gamma_1(\Delta)\Gamma_2(\Delta)=\Gamma_1(\Delta)(\Delta+\lambda_4^2),$$
$$\boldsymbol{\Theta}_{gh}(\Delta)=0\,,\boldsymbol{\Theta}_{55}(\Delta)=\boldsymbol{\Theta}_{44}(\Delta)=\Gamma_1(\Delta),m=1,2,3,4\quad g,h=1,2,3,4,5\ \ g\neq h\,. \tag{30}$$

From Eqs. (23) and (28), we find

$$\begin{aligned}\psi^* &= q_{11}(\Delta)grad\ div\boldsymbol{H}+\frac{1}{g_{12}}\Gamma_1(\Delta)\boldsymbol{H}\\ &\quad+q_{21}(\Delta)grad\,L+q_{31}(\Delta)grad\,M,\end{aligned} \tag{31}$$

$$\psi_2=q_{12}(\Delta)div\boldsymbol{H}+q_{22}(\Delta)L+q_{32}(\Delta)M, \tag{32}$$

$$\psi_3=q_{13}(\Delta)div\boldsymbol{H}+q_{23}(\Delta)L+q_{33}(\Delta)M, \tag{33}$$

where

$$q_{11}(\Delta)=\frac{1}{g_2g_{12}}(-g_1N_{11}-g_{10}N_{12})\,,\ q_{21}(\Delta)=\frac{1}{g_2g_{12}}(-g_1N_{21}-g_{10}N_{22})\,,$$

$$q_{31}(\Delta)=\frac{1}{g_2g_{12}}(-g_1N_{31}-g_{10}N_{32})\,,\ q_{12}(\Delta)=\frac{1}{g_{12}}N_{12}\,,\ q_{22}(\Delta)=\frac{1}{g_2g_{12}}N_{22}\,,$$

$$q_{32}(\Delta)=\frac{1}{g_2g_{12}}N_{32}\,,\ q_{13}(\Delta)=\frac{1}{g_2g_{12}}N_{13}\,,\ q_{23}(\Delta)=\frac{1}{g_2g_{12}}N_{23}\,,\ q_{33}(\Delta)=\frac{1}{g_2g_{12}}N_{33}\,.$$

From Eqs. (31)-(33), we have

$$\hat{\boldsymbol{\psi}}=\boldsymbol{R}^{tr}(\boldsymbol{D}_x)\boldsymbol{Q}(\boldsymbol{x}), \tag{34}$$

where

$\boldsymbol{R}^{tr}$ is the transpose of the matrix R and $\boldsymbol{R}=\left\|R_{gh}\right\|_{5\times5}$,

$$R_{mn}(\boldsymbol{D}_x)=\frac{1}{g_2}\Gamma_1(\Delta)+q_{11}(\Delta)\frac{\partial^2}{\partial x_m\partial x_n},\ R_{m5}(\boldsymbol{D}_x)=q_{13}(\Delta)\frac{\partial}{\partial x_m},\ R_{5n}(\boldsymbol{D}_x)=q_{31}(\Delta)\frac{\partial}{\partial x_n},$$

$$R_{4n}(\boldsymbol{D}_x) = q_{21}(\Delta)\frac{\partial}{\partial x_n},\ R_{55}(\boldsymbol{D}_x) = q_{33}(\Delta),\ R_{44}(\boldsymbol{D}_x) = q_{22}(\Delta).\ m,n = 1,2,3. \tag{35}$$

Also, from Eqs. (19), (29) and (34), we obtain

$$\boldsymbol{\Theta U} = \boldsymbol{R^{tr} F^{tr} U} \tag{36}$$

It implies that

$$\begin{aligned} \boldsymbol{\Theta} &= \boldsymbol{R^{tr} F^{tr}}, \\ \boldsymbol{\Theta}(\Delta) &= \boldsymbol{R}(\boldsymbol{D}_x)\boldsymbol{F}(\boldsymbol{D}_x), \end{aligned} \tag{37}$$

We assume that

$$\lambda_m^2 \neq \lambda_n^2 \neq 0, m,n = 1,2,3,4 \quad m \neq n. \tag{38}$$

Let

$$\begin{aligned} &Y(\boldsymbol{x}) = \left\|Y_{rs}(\boldsymbol{x})\right\|_{5\times 5}, Y_{mm}(\boldsymbol{x}) = \sum_{n=1}^{4} r_{1n}\zeta_n(\boldsymbol{x}), \\ &Y_{vw}(\boldsymbol{x}) = 0, \\ &m = 1,2,3,4 \ and \ v,w = 1,2,3,4,5,\ v \neq w. \end{aligned} \tag{39}$$

where

$$\zeta_n(\boldsymbol{x}) = \frac{-\exp(i\lambda_n|\boldsymbol{x}|)}{4\pi|x|}, n = 1,2,3,4. \tag{40}$$

$$r_{ml} = \prod_{\substack{m=1 \\ m\neq l}}^{4} \left(\lambda_m^2 - \lambda_l^2\right)^{-1}, l = 1,2,3,4, \tag{41}$$

$$r_{mv} = \prod_{\substack{m=1 \\ m\neq v}}^{4} \left(\lambda_m^2 - \lambda_v^2\right)^{-1}, v = 3,4. \tag{42}$$

We will prove the following lemma:

Lemma: The matrix $\boldsymbol{Y}$ defined above is the fundamental matrix of operator $\boldsymbol{\Theta}(\Delta)$, which is

$$\boldsymbol{\Theta}(\Delta)\boldsymbol{Y}(\boldsymbol{x}) = \delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}), \tag{43}$$

Proof: To prove the lemma, it is sufficient to prove that

$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(\boldsymbol{x}) = \delta(\boldsymbol{x}), \tag{44}$$

We find that

$$r_{11}+r_{12}+r_{13}+r_{14}=0,$$
$$r_{12}(\lambda_1^2-\lambda_2^2)+r_{13}(\lambda_1^2-\lambda_3^2)+r_{14}(\lambda_1^2-\lambda_4^2)=0,$$
$$r_{13}(\lambda_1^2-\lambda_3^2)(\lambda_2^2-\lambda_3^2)+r_{14}(\lambda_1^2-\lambda_4^2)(\lambda_2^2-\lambda_4^2)=0,\ r_{14}(\lambda_1^2-\lambda_4^2)(\lambda_2^2-\lambda_4^2)(\lambda_3^2-\lambda_4^2)=1,$$
$$(\Delta+\lambda_m^2)\zeta_n(\boldsymbol{x})=\delta(\vec{x})+(\lambda_m^2-\lambda_n^2)\zeta_n(\boldsymbol{x}),\ \text{m,n=1,2,3,4.} \tag{45}$$

Now consider

$$\Gamma_1(\Delta)\Gamma_2(\Delta)Y_{11}(\boldsymbol{x})=(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)$$
$$(\Delta+\lambda_4^2)\sum_{n=1}^{4}r_{1n}\left[\delta+(\lambda_1^2-\lambda_n^2)\zeta_n\right],$$
$$=(\Delta+\lambda_2^2)(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)\sum_{n=2}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)\zeta_n,$$
$$=(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)\sum_{n=2}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)\left[\delta+(\lambda_2^2-\lambda_n^2)\zeta_n\right]$$
$$=(\Delta+\lambda_3^2)(\Delta+\lambda_4^2)\sum_{n=3}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)(\lambda_2^2-\lambda_n^2)\zeta_n\ ,$$
$$=(\Delta+\lambda_4^2)\sum_{n=3}^{4}r_{1n}(\lambda_1^2-\lambda_n^2)(\lambda_2^2-\lambda_n^2)\ (\lambda_3^2-\lambda_n^2)\left[\delta+(\lambda_3^2-\lambda_n^2)\zeta_n\right]\quad=(\Delta+\lambda_4^2)\zeta_n=\delta\ , \tag{46}$$

We introduce the matrix

$$\boldsymbol{G}(\boldsymbol{x})=\boldsymbol{R}(\boldsymbol{D}_x)\boldsymbol{Y}(\boldsymbol{x}), \tag{47}$$

From Eqs. (31)-(33), (37) and (43), we obtain

$$\begin{aligned}\boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{G}(\boldsymbol{x})&=\boldsymbol{F}(\boldsymbol{D}_x)\boldsymbol{R}(\boldsymbol{D}_x)\boldsymbol{Y}(\boldsymbol{x})\\&=\boldsymbol{\Theta}(\Delta)\boldsymbol{Y}(\boldsymbol{x})=\delta(\boldsymbol{x})\boldsymbol{I}(\boldsymbol{x}).\end{aligned} \tag{48}$$

Hence $\boldsymbol{G}(\boldsymbol{x})$ is the solution of Eq. (21).

Therefore we have proved the following theorem:

Theorem 1. If the condition (14) is satisfied, then the matrix $\boldsymbol{G}(\boldsymbol{x})$ (which is constructed using four elementary functions ζ_1,ζ_2,ζ_3 *and* ζ_4 in Eq. (40)) defined by Eq. (47) is a solution of system of Eqs. (9)-(11), where $\boldsymbol{R}(\boldsymbol{D}_x)$ and $\boldsymbol{Y}(\boldsymbol{x})$ are given by Eqs. (35) and (39) respectively.

Now we can establish the basic properties of $\boldsymbol{G}(\boldsymbol{x})$. Theorem 1 leads to the following results.

Corollary 1. If the condition (14) is satisfied, then the fundamental solution of the system

$$g_2\Delta\boldsymbol{u}+g_1\nabla div\,\boldsymbol{u}=0, \tag{49}$$

$$g_{12}\Delta T=0, \tag{50}$$

$$\frac{g_8}{\tau}T+\Delta N-g_7N=0, \tag{51}$$

is the matrix $\Phi = \left\| \Phi_{gh} \right\|_{5\times 5}$, where

$$\Phi_{mn}(\boldsymbol{x}) = \left(g_2 \Delta \delta_{mn} + g_1 \frac{\partial^2}{\partial x_m \partial x_n} \right) \zeta^3(x)$$

$$,\Phi_{m4}(\boldsymbol{x}) = 0, \Phi_{4n}(\boldsymbol{x}) = 0, \Phi_{44}(\boldsymbol{x}) = g_{12}\zeta^4(x),$$

$$\Phi_{55}(\boldsymbol{x}) = g_7 \zeta^4(x), \Phi_{45}(\boldsymbol{x}) = 0 ,$$

$$\Phi_{54}(\boldsymbol{x}) = \frac{g_8}{\tau}\zeta^4(x),$$

$$\Phi_{\boldsymbol{mn}}(\boldsymbol{x}) = \mathrm{O}(|\boldsymbol{x}|^{-1}) \quad \text{and} \quad \Phi_{\boldsymbol{mn},r}(\boldsymbol{x}) = \mathrm{O}(|\boldsymbol{x}|^{-2})$$

hold in a neighbourhood of the origin, where m,n =1,2,3,4,5. and r =1,2,3.

On the basis of Theorem 1 and Corollary 1 we obtain the following

Theorem 2. If the condition (14) is satisfied, then the relations

$$\boldsymbol{G}_{\boldsymbol{mn}}(\boldsymbol{x}) = \mathrm{O}(|\boldsymbol{x}|^{-1}) \quad \text{and} \quad \boldsymbol{G}_{\boldsymbol{mn},r}(\boldsymbol{x}) = \mathrm{O}(|\boldsymbol{x}|^{-2})$$

$$\boldsymbol{G}_{\boldsymbol{mn}}(\boldsymbol{x}) - \Phi_{\boldsymbol{mn}}(\boldsymbol{x}) = const + \mathrm{O}(|\boldsymbol{x}|)$$

$$\frac{\partial^q}{\partial x_1^{q_1} \partial x_2^{q_2} \partial x_3^{q_3}} \left[\boldsymbol{G}_{\boldsymbol{mn}}(\boldsymbol{x}) - \Phi_{\boldsymbol{mn}}(\boldsymbol{x}) \right] = \mathrm{O}(|\boldsymbol{x}|^{1-q})$$

hold in a neighbourhood of the origin, where $q = q_1 + q_2 + q_3, q \geq 1, q_r \geq 0, r = 1,2,3$ *and* $m,n = 1,2,3,4,5.$ Thus , $\Phi(\boldsymbol{x})$ is the singular part of the fundamental matrix $\boldsymbol{G}(\boldsymbol{x})$ in the neighbourhood of the origin. Taking into account inequality $\mathrm{Im}\,\lambda_m > 0 (m = 1,2,3,4)$ we have

$$\zeta_n(\boldsymbol{x}) = \exp(-\lambda_0 |\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-1}) \qquad \textit{and} \qquad \zeta_{n,r}(\boldsymbol{x}) = \exp(-\lambda_0 |\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-2})$$

for $|\boldsymbol{x}| >> 1,$ where $\lambda_0 = \min\left\{ \mathrm{Im}\,\lambda_j , j = 1,2,3,4 \right\} > 0$ *and* $r = 1,2,3.$ Consequently, on the basis of Theorem 1 each element of $\boldsymbol{G}(\boldsymbol{x})$ is represented in the form

$$\Phi_{mn}(\boldsymbol{x}) = \sum_{s=1}^{4} \Phi_{mn}^{(s)}(\boldsymbol{x}),$$

where $\left(\Delta + \lambda_s^2\right)\Phi_{mn}^{(s)}(\boldsymbol{x}) = 0$ *for* $|x| \neq 0,$ and has the following property at the infinity

$$\Phi_{mn}^{(s)}(\boldsymbol{x}) = \exp(-\lambda_0 |\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-1}), \quad \textit{and} \quad \Phi_{mn,r}^{(s)}(\boldsymbol{x}) = \exp(-\lambda_0 |\boldsymbol{x}|)\mathrm{O}(|\boldsymbol{x}|^{-2}),$$

for $|\boldsymbol{x}| >> 1, m,n = 1,2,3,4,\ s = 1,2,3,4$

and $r = 1,2,3.$

5. Plane waves

We consider a plane wave propagation in a homogeneous isotropic photothermoelastic medium under Moore-Gibson-Thompson thermoelasticity. For two dimensional problem, we take

$$u_i = \big(u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t)\big), \\ T(x_1, x_3, t), N(x_1, x_3, t). \tag{52}$$

By Helmholtz decomposition theorem, we have

$$u_1 = \frac{\partial \Phi}{\partial x_1} - \frac{\partial \Psi}{\partial x_3} \text{ and } u_3 = \frac{\partial \Phi}{\partial x_3} + \frac{\partial \Psi}{\partial x_1} \quad . \tag{53}$$

Eqs. (5)-(7), with the aid of Eqs. (52) and (53), take the form

$$\left(\Delta - \frac{\partial^2}{\partial t^2}\right)\Phi - T - g_3 N = 0, \tag{54}$$

$$\left(\Delta - \frac{1}{g_2}\frac{\partial^2}{\partial t^2}\right)\Psi = 0, \tag{55}$$

$$\left[g_5\left(1+\tau_o \frac{\partial}{\partial t}\right)\Delta\left(\frac{\partial^2}{\partial t^2}\right)\right]\Phi - \\ \left[\Delta\frac{\partial}{\partial t} + g_4\Delta - \left(1+\tau_o\frac{\partial}{\partial t}\right)\frac{\partial^2}{\partial t^2}\right]T \\ -\left[\frac{g_6}{\tau}\left(1+\tau_o\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\right]N = 0, \tag{56}$$

$$g_8\frac{T}{\tau} + \left(\Delta - g_7\left(\frac{\partial}{\partial t} + \frac{1}{\tau}\right)\right)N = 0, \tag{57}$$

We assume the solution for Eqs. (54)-(57) of the form

$$(\Phi, \Psi, T, N) = (\overline{\Phi}, \overline{\Psi}, \overline{T}, \overline{N}) \\ e^{-i\left[\xi(l_1 x_1 + l_3 x_3) - \omega t\right]} \tag{58}$$

where $\omega = \xi c$ I the frequency, ξ is the wave number and c is the phase velocity. $\overline{\Phi}, \overline{\Psi}, \overline{T}, \overline{N}$ are undetermined amplitudes, that are dependent on time t and coordinates $x_m (m = 1,3)$. l_1 *and* l_3 are the direction cosines of the wave normal to the $x_1 x_3$ – plane with the property $l_1^2 + l_3^2 = 1$.

Making use of Eq. (58) in Eqs. (54)-(57), we get

$$\left(-\xi^2 + \omega^2\right)\overline{\Phi} - \overline{T} - g_3\overline{N} = 0, \tag{59}$$

$$g_{10}\xi^2\overline{\Phi} + \left(g_{12}\xi^2 - \omega^2 g_9\right)\overline{T} - \frac{g_{13}}{\tau}\overline{N} = 0, \tag{60}$$

$$\frac{g_8}{\tau}\overline{T}+\left(-\xi^2+\frac{g_{14}}{\tau}\right)\overline{N}=0, \tag{61}$$

$$\left(\xi^2-\frac{\omega^2}{g_2}\right)\overline{\Psi}=0, \tag{62}$$

For non- trivial solution of the system of Eqs. (59)-(61), yields the following polynomial characteristic equation in ξ as

$$\left(\xi^6+R_1\xi^4+R_2\xi^2+R_3\right)=0, \tag{63}$$

where

$$R_1=\frac{\left(-g_{10}\tau-g_9\omega^2\tau-g_{12}\omega^2\tau-g_{12}g_{14}\right)}{g_{12}\tau},$$

$$R_2=\frac{\begin{pmatrix}-g_8g_{13}+\omega^4g_9\tau^2+g_{10}g_{14}\tau+g_9g_{14}\omega^2\tau\\-g_3g_{10}g_8\tau-g_{12}g_{14}\omega^2\tau\end{pmatrix}}{g_{12}\tau^2},\ R_3=\frac{\left(\omega^2g_8g_{13}-\omega^2\tau g_8g_{14}\right)}{g_{12}\tau^2}.$$

Solving Eq. (63), we obtain six roots of ξ, that is $\xi_1,\xi_2 and \xi_3$ correspond to positive x_3 direction and other three roots $-\xi_1,-\xi_2 and -\xi_3$ correspond to negative x_3 direction. Corresponding to roots $\xi_1,\xi_2 and\ \xi_3$, there exist three waves in descending order of their velocity, namely a longitudinal wave (P-wave) , thermal wave (T-wave) and plasma wave(PL-wave). From Eq. (62) we obtain two roots of ξ, that is $\pm\xi_4$ and corresponding to this root ,there exists a transverse wave (SV).It is noticed that these two values are unaffected by the thermal properties of the photothermoelastic medium.

We derive the expressions of phase velocity, attenuation coefficient, specific loss and penetration depth of these type of waves as

(i) Phase velocity

The phase velocities is given by

$$V_i=\frac{\omega}{\left|\mathrm{Re}(\xi_i)\right|},\ i=1,2,3. \tag{64}$$

where V_1,V_2,V_3 are the phase velocities of P,T and plasma waves respectively.

(ii) Attenuation coefficient

The attenuation coefficient are defined as

$$Q_i=\mathrm{Im}(\xi_i),\ i=1,2,3. \tag{65}$$

where $Q_1,Q_2\ and\ Q_3$ are the attenuation coefficients of P,T and plasma waves respectively.

(iii) Specific loss

The specific loss is defined as

$$R_i = \left(\frac{\Delta W}{W}\right) = 4\pi \left|\frac{\mathrm{Im}(R_i)}{\mathrm{Re}(R_i)}\right|, \quad i = 1,2,3. \tag{66}$$

where W is elastic energy and $R_1, R_2 \; and \; R_3$ are specific loss of P,T and plasma waves respectively.

(iv) Penetration depth

The penetration depth is defined as

$$S_i = \frac{1}{|\mathrm{Im}(\xi_i)|}, \quad i = 1,2,3. \tag{67}$$

where $S_1, S_2 \; and \; S_3$ are penetration depth of P,T and plasma waves respectively.

6. Particular cases

Photothermoelasticity under Moore–Gibson–Thompson model in which $\mathrm{K}, K^* and \tau_o$ all are positive is limited to following cases

(i) If we take $K^* = 0$ in Eqs. (47) and (63), we obtained the corresponding result for Lord and Shulman's (LS) model.

(ii) If we take $\tau_o = \mathrm{K} = 0$ in Eqs. (47) and (63), we obtain the corresponding result for Green and Naghdi of type-II (GN-II) model.

(iii) If we take $\tau_o = 0$ in Eqs. (47) and (63), we obtain the corresponding result for Green and Naghdi of type-III (GN-III) model.

7. Numerical results and discussion

For the numerical calculations we take material constants for an isotropic Silicon (Si) material as $\lambda = 3.64 \; N/m^2$, $\mu = 5.46 \; N/m^2$, $\alpha_t = 0.00414 K^{-1}, \alpha_n = -0.00198 m^3/kg$, $\rho = 2330 kg/m^3$, $T_o = 300K$, $K = 150 w/mk, E_g = 1.11eV, C_e = 695 \; j/kg\,K, \tau = 0.05s$, $D_e = 2.5 m^2/s, n_o = 10^2 m^{-3}$

The values of phase velocity, attenuation coefficient, specific loss and penetration depth of plane waves are determined by using MATLAB software. The variations of phase velocity, attenuation coefficient, specific loss and penetration depth with respect to frequency are shown in Figs. (1.1) to (1.12) respectively. Comparison has been made among the generalization theories presented by Moore-Gibson-Thomson thermoelasticity (MGTE), Lord and Shulman (LS), Green and Naghdi of type-II (GN-II) and Green and Naghdi of type-III (GN-III).

In all the figures solid line correspond to photothermoelastic MGTE model, dashed line corresponds to LS model, dotted line corresponds to GN-II model and dashed-dot line corresponds to GN-III model.

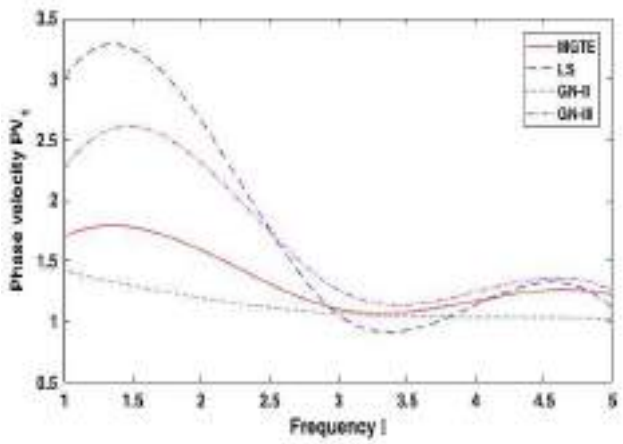

Fig. 1.1 Profile of phase velocity PV_1 vs. ω

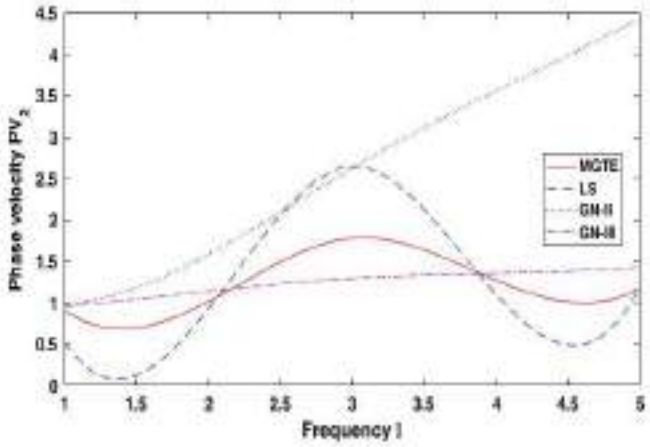

Fig. 1.2 Profile of phase velocity PV_2 vs. ω

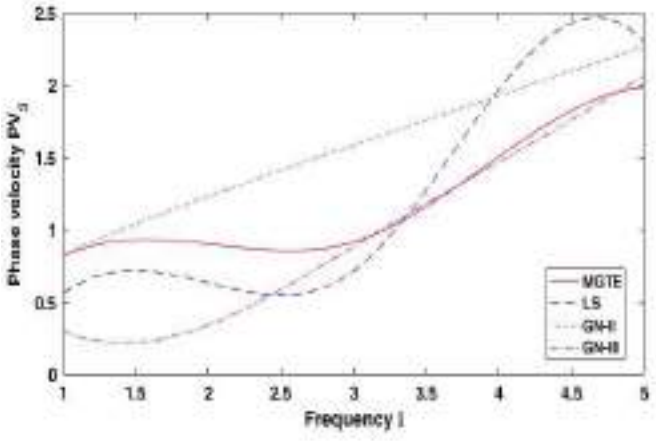

Fig. 1.3 Profile of phase velocity PV_3 vs. ω

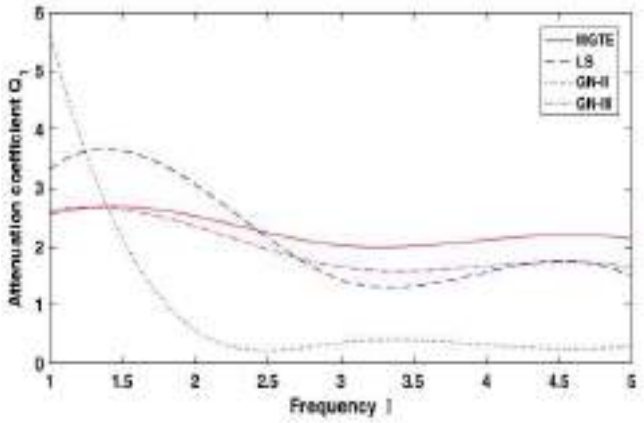

Fig. 1.4 Profile of attenuation coefficient Q_1 vs. ω

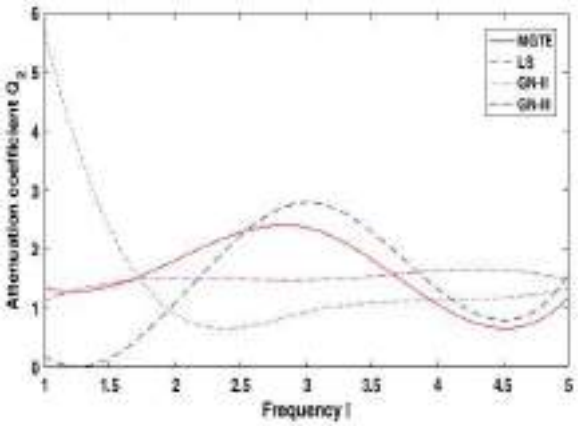

Fig. 1.5 Profile of attenuation coefficient Q_2 vs. ω

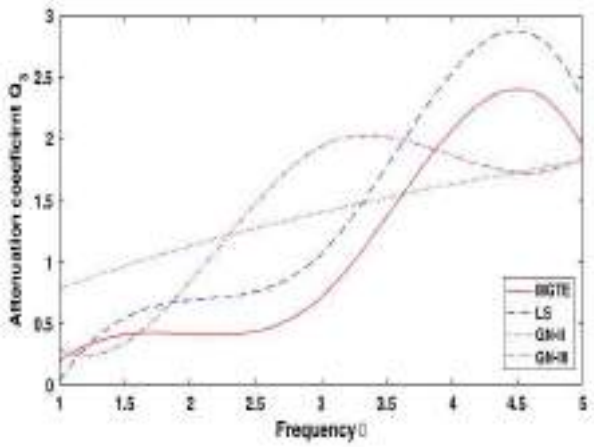

Fig. 1.6 Profile of attenuation coefficient Q_3 vs. ω

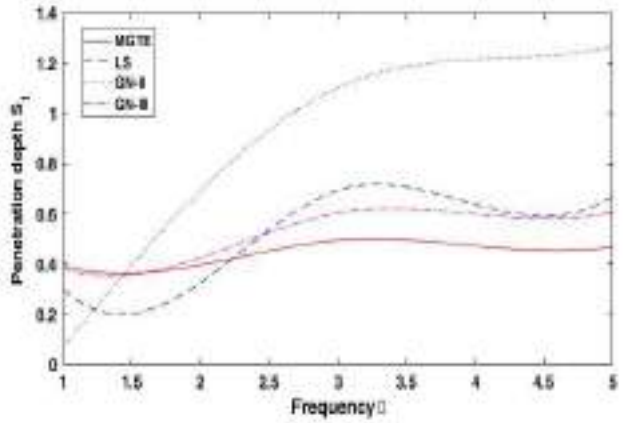

Fig. 1.7 Profile of penetration depth S_1 **vs.** ω

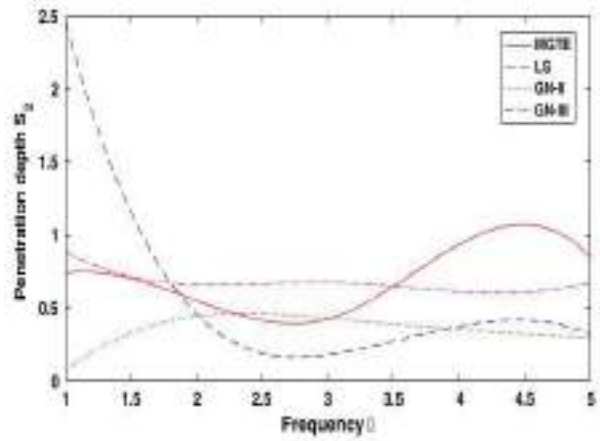

Fig. 1.8 Profile of penetration depth S_2 vs. ω

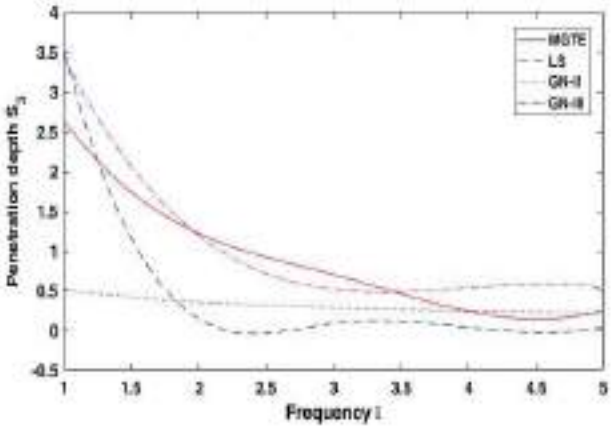

Fig. 1.9 Profile of penetration depth S_3 vs. ω

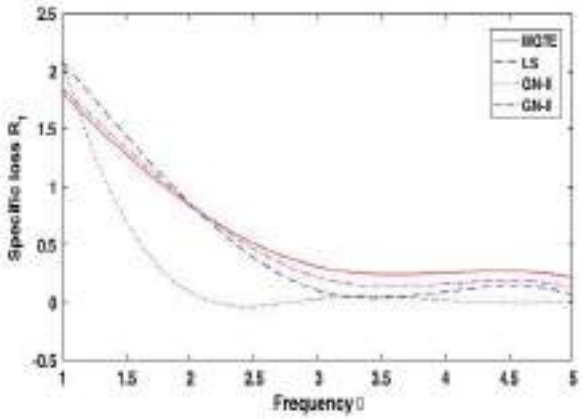

Fig. 1.10 Profile of specific loss R_1 vs. ω

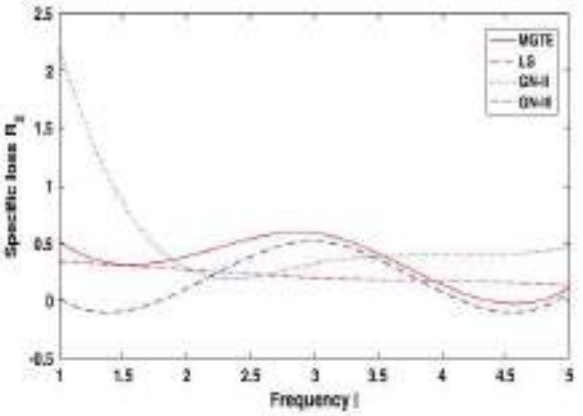

Fig. 1.11 Profile of specific loss R_2 vs ω

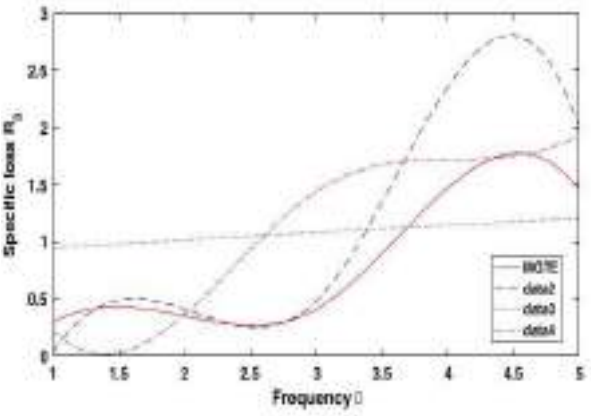

Fig. 1.12 Profile of specific loss R_3 vs. ω

Phase velocity

Fig. 1.1 depicts trend of phase velocity PV_1 vs. ω. Initially, the magnitude of PV_1 is maximum for LS model and minimum for GN-II model for the lower frequency. The behaviour and variation of PV_1 for MGTE, LS and GN-III is oscillatory. GN-III model minimize the value of PV_1 for extreme values of frequency. All the curves correspond to PV_1 are in decreasing trend for the whole range of ω.

Fig. 1.2 displays trend of phase velocity PV_2 vs. ω. In the initial range of frequency, the magnitude of PV_2 is higher for GN-III model and lower for LS model. The value of PV_2 is monotonically increasing with the increase in frequency due to GN-II model. The curves correspond to PV_2 is oscillatory in behaviour for LS and MGTE model.

Fig. 1.3 demonstrates trend of phase velocity PV_3 vs. ω. The magnitude of PV_3 is maximum for lower frequency due to GN-II model. The values of PV_3 is monotonically increasing for the whole range of frequency due to GN-II and GN-III models. The curves correspond to PV_3 is oscillatory in behaviour for LS and MGTE model.

Attenuation coefficient

Fig. 1.4 depicts trend of attenuation coefficient Q_1 vs. ω. In the initial range of frequency, the magnitude of Q_1 is maximum for GN-II model and minimum due to MGTE model. The behaviour and variation of Q_1 is opposite oscillatory for GN-III and LS model, in the range $2.5 \le \omega \le 4.5$. The values of Q_1 is monotonically decreasing in the range $0 \le \omega \le 2$ due to GN-II model as compare to other models.

Fig. 1.5 demonstrates trend of attenuation coefficient Q_2 vs. ω. For the extreme values of frequency, the magnitude of Q_2 is maximum for GN-II model and minimum due to LS model, whereas for intermediate values of frequency LS model intensify and GN-II model minimize the values of Q_2. The curves correspond to Q_2 are oscillatory in nature due to MGTE, LS and GN-III.

Fig. 1.6 displays trend of attenuation coefficient Q_3 vs. ω. In the initial range of frequency, the magnitude of Q_3 is maximum for GN-II model and minimum due to LS model. The behaviour and variation of Q_3 is opposite oscillatory for GN-III and LS model, for the whole range of frequency. The curves correspond to Q_3 for MGTE and LS model fluctuate in the same way.

Penetration depth

Fig. 1.7 displays trend of penetration depth S_1 vs. ω. Initially, the magnitude of S_1 is maximum for MGTE model and minimum due to GN-II model. The behaviour and variation of S_1 is monotonically increasing due to GN-II model for whole range of frequency. All the curves correspond to S_1 are in increasing trend for higher values of frequency.

Fig. 1.8 depicts trend of penetration depth S_2 vs. ω. Initially, the magnitude of S_2 is maximum for LS model and minimum due to GN-II model. The behaviour and variation of S_2 is monotonically decreasing for LS model in the range $0 \le \omega \le 2.5$ and opposite oscillatory for GN-III & MGTE model and LS & GN-II model.

Fig. 1.9 demonstrates trend of penetration depth S_3 vs. ω. Initially, the magnitude of S_3 is maximum for LS model and minimum due to GN-II model. The behaviour and variation of S_3 is decreasing for all models. The variation of S_3 is opposite oscillatory for GN-III and MGTE model

for the whole range of frequency.

Specific loss

Fig. 1.10 demonstrates trend of specific loss R_1 vs. ω. Initially, the magnitude of R_1 is maximum for LS model and minimum due to MGTE model. The behaviour and variation of R_1 is decreasing for all models for the whole range of frequency. The variation of R_1 is monotonically decreasing for GN-II model in the range $0 \leq \omega \leq 2$. The curves due to MGTE,LS and GN-III travels in similar manner with small difference in their magnitude.

Fig. 1.11 displays trend of specific loss R_2 vs. ω. Initially, the magnitude of R_2 is maximum for GN-II model and minimum due to LS model. The behaviour and variation of R_2 is oscillatory for all models except GN-III model. The variation of R_2 is opposite oscillatory for GN-II and MGTE models in the range $\omega > 2$.

Fig. 1.12 depicts trend of specific loss R_3 vs. ω. Initially, the magnitude of R_3 is maximum for GN-II model and minimum due to LS model. The behaviour and variation of R_3 is oscillatory for all models except GN-II model. The variation of R_3 is opposite oscillatory for GN-III and LS models for the whole range of frequency. The curve due to MGTE model behave opposite oscillatory with GN-III model in the initial range of frequency.

8. Conclusions

Study of fundamental solution and plane wave vibrations are an important problem of mechanics of continua.The fundamental solution of system of equations in the generalized theories of photothermoelastic medium under Moore-Gibson-Thompson thermoelasticity for steady oscillation in terms of elementary functions has been constructed.

On the basis of fundamental solution of the Eqs. (9)-(11), it is possible to construct the surface (single layer and double layer) and volume potential in photothermoelastic with MGTE model and to establish their basic properties. To obtain the formulae of integral representation of regular solutions of the Eqs. (9)-(11), for the investigation of three dimensional BVP in the considered model by means of potential method and the theory of two dimensional singular integral equation. The method of fundamental solution is an elegant method for the solution of the basic differential equations. On the basis of the Theorem 1 and Theorem 2 discussed above, we can construct the regular solution of the 3 dimensional BVP of steady vibration by using potential methods and theory of singular integral equation in the considered model.

The propagation of plane wave in the considered medium under Moore-Gibson-Thompson thermoelasticity has also been studied. It is observed that there exist three longitudinal waves namely Longitudinal wave (P-wave), thermal wave(T-wave) and plasma wave(PL-waves) in addition to transverse wave SV wave which is not affected by thermal properties and photothermal effect of the materials.

Opposite behaviour of phase velocities PV_1 and PV_2 is observed for lower frequency due to all models. For higher frequency, PV_3 attain higher magnitude due to one relaxation time in comparison to other models. Attenuation coefficient correspond to P-wave and T-wave remains higher and lower for higher frequency respectively in MGTE model in comparison to other models. One relaxation

time predominant attenuation coefficient corresponds to PL-wave in comparison to other assumed models. Without energy dissipation and one relaxation time has predominant impact on S_1 and $S_2$for higher and lower frequency respectively. S_3 has dominant impact of energy dissipation for lower frequency. Impact of MGTE on R_1 is observed stronger in comparison to R_2 and R_3 for higher frequency.

The result showed that several physical quantities like MGTE, photothermoelasticity significantly impact the system interaction. The comparison of the different models on attributes of the waves shows the dependence of physical field quantities. The result shows that MGTE model of photothermoelastic predicts a finite speed of wave propagation that makes a new model more consistent with the physical property of the material. The model explored in this work find application in various field such as structural engineering, theoretical seismology etc.

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